10. DERIBATUEN APLIKAZIOAK. (x₀ emanda)

279 or. [1] a) $y = \dfrac{5x^3+7x^2-16x}{x-2}$ $x=0$, $x=1$, $x=3$ puntuetan.

Ukitzailearen ekuazioa $\boxed{y = y_0 + m(x-x_0)}$

edo $\boxed{y = f(x_0) + f'(x_0)\cdot(x-x_0)}$

- Ekuazioan ordezkatzeko puntua $P(x_0, y_0)$ eta $f'(x_0)$ behar da:
- Deribatua: $y' = \dfrac{(15x^2+14x-16)(x-2)-(5x^3+7x^2-16x)}{(x-2)^2}$

$$y' = \frac{15x^3-30x^2+14x^2-28x-16x+32-5x^3-7x^2+16x}{(x-2)^2} =$$

$$= \frac{10x^3-23x^2-28x+32}{(x-2)^2}$$

- x_0 bakoitzarentzako y_0 eta $f'(x_0)$ kalkulatu:

* $x=0 \to f(0)=0 \to \boxed{P_1(0,0)}$ $m=f'(0)=\frac{32}{4}=\boxed{8}$

* $x_1=1 \to f(1)=\dfrac{5\cdot1+7\cdot1-16\cdot1}{1-2}=4 \to \boxed{P(1,4)}$

$$f'(1)=\frac{10\cdot1-23-28+32}{(1-2)^2}=\boxed{-9}$$

* $x_2=3 \to f(3)=\dfrac{5\cdot3^3+7\cdot3^2-16\cdot3}{3-2}=150 \to \boxed{P(3,150)}$

$$f'(3)=\frac{10\cdot3^3-23\cdot3^2-28\cdot3+32}{(3-2)^2}=\boxed{11}$$

UKITZAILEAK

$y_1 = 0+8(x-0)$
$y_2 = 4-9(x-1)$
$y_3 = 150+11(x-3)$

$\Longrightarrow$

$$\boxed{\begin{aligned} y_1 &= 8x \\ y_2 &= -9x+13 \\ y_3 &= 11x+117 \end{aligned}}$$

9.2

271 b) $x^2+y^2-2x+4y-24=0$. (Inplizituak)

$x_0=3$

Zuzen ukitzailearen ekuazioarako $P(x_0, y_0)$ eta $f'(x_0)=m$ behar da.

• PUNTUA KALKULATZEKO

$$3^2+y^2-2\cdot3+4\cdot y-24=0$$
$$9+y^2-6+4y-24=0$$
$$y^2+4y-21=0$$
$$y=\frac{-4\pm\sqrt{4^2-4(-21)}}{2}=\begin{cases} y_1=3 \\ y_2=-7 \end{cases}$$

$$\boxed{\begin{matrix} P_1(3,3) \\ P_2(3,-7) \end{matrix}}$$

• DERIBATUA (inplizituak)

$$2x+2yy'-2+4y'=0$$
$$y'(2y+4)=2-2x$$
$$y'=\frac{2-2x}{2y+4} \Longrightarrow \boxed{y'=\frac{1-x}{y+2}}$$

• MALDA → DERIBATUA PUNTUAN DA

$$P_1(3,3) \longrightarrow y'=\frac{1-3}{3+2}=\boxed{\frac{-2}{5}}$$

$$P_2(3,-7) \longrightarrow y'=\frac{1-3}{-7+2}=\boxed{\frac{2}{5}}$$

• UKITZAILEAK $\boxed{y=y_0+m(x-x_0)}$

$$y_1=3+\left(\frac{-2}{5}\right)(x-3) \longrightarrow y_1=\frac{-2}{5}x+\frac{21}{5}$$

$$y_2=-7+\frac{2}{5}(x-3) \longrightarrow y_2=\frac{2}{5}x-\frac{41}{5}$$

c) $y=\frac{x^3}{3}-x^2+3x-6.$ (m emanda) 9.3.

$y-x=9$. zuzenarekiko paraleloa.

- Ukitzailea $y-x=9$ zuzenarekiko paraleloa bada, malda bardina izango dabe

$$y-x=9 \longrightarrow y=9+x \longrightarrow \boxed{m=1}$$

- Malda, funtzioaren deribatua puntuan da, beraz; funtzioa deribatuko da, eta $m=1$-ekin bardindu.

$$\boxed{f'(x_0)=m}$$

$$y=\frac{x^3}{3}-x^2+3x-6$$

$$y'=f'(x)=\frac{\cancel{3}x^2}{\cancel{3}}-2x+3$$

- Bardintzen da $m=1$.

$$x^2-2x+3=1$$

$$x^2-2x+2=0$$

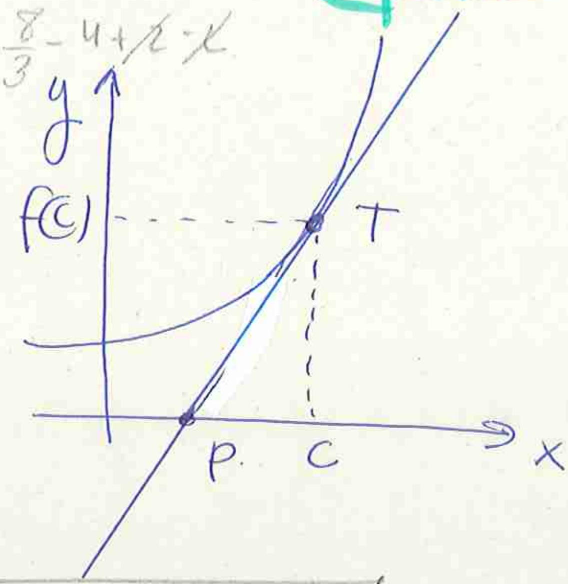
$$x=\frac{2\pm\sqrt{2^2-4\cdot 2}}{2}=\frac{2\pm\sqrt{-4}}{2}$$

$\nexists x$, ez dago punturik non ukitzailea $y-x=9$ zuzenaren paraleloa dan.

9.4

d) $y = \frac{x^3}{3} - x^2 + x - 2$ $P(2,0)$ (kanpoko puntua)

$\frac{8}{3} - 4 + \cancel{2} - \cancel{2}$



· $P(2,0)$ · kanpoko puntua da

Bi puntuen arteko malda, T eta P, eta deribatua T puntuan berdinak dira

① Malda planteatu m_{TP}

$m = \frac{\Delta y}{\Delta x}$

$P(2,0)$

$T(c, f(c))$

$f(c) = \frac{c^3}{3} - c^2 + c - 2$

$$\boxed{m = \frac{\frac{c^3}{3} - c^2 + c - 2 - 0}{c - 2}}$$

② Deribatua T puntuan

$f'(x) = \frac{\cancel{3}x^2}{\cancel{3}} - 2x + 1$

$\boxed{f'(c) = c^2 - 2c + 1}$

③ Berdindu $\boxed{m = f'(c)}$

$\frac{\frac{c^3}{3} - c^2 + c - 2}{c - 2} = c^2 - 2c + 1$

$\frac{c^3}{3} - c^2 + \cancel{c} - \cancel{2} = c^3 - 2c^2 + \cancel{c} - 2c^2 + 4c - \cancel{2}$

$\frac{c^3}{3} - c^2 = c^3 - 4c^2 + 4c$

$c^3 = 3c^3 - 9c^2 + 12c \longrightarrow$

$2c^3 - 9c^2 + 12c = 0$

$c(2c^2 - 9c + 12) = 0$

$c = \frac{9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot 12}}{2 \cdot 2} = \nexists$

$\boxed{c = 0}$

T puntua $\rightarrow T(c, f(c))$

$\boxed{T(0, -2)}$

$f'(0) = 0^2 - 0c + 1 = 1 \longrightarrow \boxed{m = 1}$

④ Zuzen ukitzailea

$\boxed{y = y_0 + m(x - x_0)}$

$y = -2 + 1(x - 0)$

$\boxed{y = x - 2}$

9.S

288) 4 HAZKUNDE - TARTEAK (ariketa ebatzia)

Aztertu HAZKUNDEA, MAX, MIN

$$f(x) = e^x \cdot (x^2 - 3x + 1)$$

Funtzioa jarraia eta deribagarria da $\mathbb{R}$ bere definizio eremu osoan.

• Hazkundea aztertzeko $f'(x)$: $\begin{cases} f'(x) > 0 & \text{GORAK} \\ f'(x) < 0 & \text{BEHERA.} \end{cases}$

$$f'(x) = e^x \cdot (x^2 - 3x + 1) + e^x (2x - 3) =$$

$$f'(x) = e^x (x^2 - x - 2)$$

• $f'(x) = 0$ puntuak bilatzen ditugu.

$$\underbrace{e^x}_{0} \cdot \underbrace{(x^2 - x - 2)}_{0} = 0. \quad \begin{cases} e^x \neq 0 \\ x^2 - x - 2 = 0 \end{cases}$$

$$(x-2)(x+1) = 0 \quad \begin{cases} x_1 = -1 \\ x_2 = 2 \end{cases}$$

		-1		2	
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$
$f'(x)$	$+$	max	$-$	min	$+$

G. TARTEA $(-\infty, -1) \cup (2, +\infty)$

B. TARTEA $(-1, 2)$

max $(-1, 5/e)$ $\quad f(-1) = e^{-1} \cdot 5 \cdot 5/e$

min $(2, -e^2)$ $\quad f(2) = e^2(-1)$

288 4b Ariketa ebatziak

b) $f(x) = \begin{cases} -x^2-2x & x \leq 0 \\ x\ln x & x>0 \end{cases}$

Jarraitasuna $x=0$

$f(0)=0$

$\left.\begin{array}{l} \lim\limits_{x\to 0^-} (-x^2-2x)=0 \\ \lim\limits_{x\to 0^+} (x\ln x)=0. \end{array}\right\}$

$f(0)=\lim\limits_{x\to 0} f(x)$

Jarraia $\mathbb{R}$ osoan.

Deribagarritasuna.

$f'(x) = \begin{cases} -2x-2 & x<0 \\ 1+\ln x & x>0. \end{cases}$

$\left.\begin{array}{l} f'(0^-) = -2\cdot 0-2 = -2 \\ f'(0^+) = 1+\ln 0 = 1 \end{array}\right\}$ Ez da deribag. $x=0$

Hazkundea

Deribatua nulua zein den aztertu: $f'(x)=0$

$\left\{\begin{array}{l} -2x-2=0 \longrightarrow x=-1. \quad x<0 \\ 1+\ln x=0 \longrightarrow \ln x=-1 \\ \qquad\qquad\qquad\quad x=e^{-1} \quad x>0. \end{array}\right.$

		-1		0		1/e	
$f(x)$	↗		↘		↘		↗
$f'(x)$	+	MAX	−		−	MIN	+

G. TARTEA $(-\infty,1) \cup (1/e,+\infty)$

B. TARTEA $(-1,0) \cup (0,1/e)$

MAX $(-1,1)$

MIN $(1/e,-1/e)$

29.om. Zuzen ukitzaileen ekuazioak

1a) $y = \ln(\mathrm{tg}\, 2x) \quad x = \pi/8.$

Zuzen ukitzailea $y = y_0 + m(x - x_0)$
edo $y = f(x_0) + f'(x_0)\cdot(x - x_0)$

- $P(x_0, y_0)$
  $x = \pi/8 \rightarrow f(\pi/8) = \ln \underbrace{\mathrm{tg}\left(\frac{\pi}{8}\cdot 2\right)}_{1} = 0 \rightarrow \boxed{P\left(\frac{\pi}{8}, 0\right)}$

- Maldo = Deribatua $x = \pi/8$ danean:
  $f'(x) = \frac{1}{\mathrm{tg}\, 2x} \cdot 2\cdot(1 + \mathrm{Tg}^2(2x))$
  $f'(\pi/8) = \frac{1}{\underbrace{\mathrm{tg}(\pi/8\cdot 2)}_{1}} \cdot 2\left(1 + \underbrace{\mathrm{tg}^2\left(2\cdot\frac{\pi}{8}\right)}_{1}\right) = 4$

- ukitzailea
  $P(\pi/8, 0) \longrightarrow y = 0 + 4(x - \pi/8) \longrightarrow \boxed{y = 4x - \frac{\pi}{2}}$
  $m = 4.$

b) $y = \sqrt{\sin 5x} \quad x_0 = \pi/6.$

- $P(x_0 y_0)$
  $x_0 = \pi/6$
  $f(\pi/6) = \sqrt{\sin\frac{5\pi}{6}} = \sqrt{\sin 150} = \sqrt{\sin 30} =$
  $= 1/\sqrt{2} = \sqrt{2}/2$

  $\boxed{P\left(\frac{\pi}{6}, \frac{\sqrt{2}}{2}\right)}$

- Maldo $f'(x) = \frac{1}{2\sqrt{\sin 5x}} \cdot \cos(5x)\cdot 5 = \frac{5\cos(5x)}{2\sqrt{\sin(5x)}}$

  $f'\left(\frac{\pi}{6}\right) = \frac{5}{2}\cdot\frac{\cos(5\pi/6)}{\sqrt{\sin(5\pi/6)}} = \frac{5}{2}\frac{\cos 150}{\sqrt{\sin 150}} =$

  $= \frac{5(-\cos 30^\circ)}{2\sqrt{\sin 30}} = \frac{5(-\sqrt{3}/2)}{2\sqrt{1/2}} = \frac{-5\sqrt{3}}{2\sqrt{2}} = \boxed{-\frac{5\sqrt{6}}{4}}$

Ukitzailea $\boxed{y = \frac{\sqrt{2}}{2} - \frac{5\sqrt{6}}{4}\left(x - \pi/6\right)}$

c) $x^2+y^2-2x-8y+15=0 \quad x_0=2$ INPLIZIT. 8

- $\underline{P(x_0,y_0)}$

$2^2+y^2-2\cdot 2-8y+15=0$

$\cancel{4}+y^2-\cancel{4}-8y+15=0$

$y^2-8y+15=0.$

$y=\dfrac{8\pm\sqrt{8^2-4\cdot 15}}{2}= \begin{cases} y_1=5 \\ y_2=3. \end{cases}$

$\boxed{\begin{matrix} P_1(2,5) \\ P_2(2,3) \end{matrix}}$

- Deribatua

$2x+2yy'-2-8y'=0$

$y'(2y-8)=2-2x$

$y'=\dfrac{2-2x}{2y-8}$

$\boxed{y'=\dfrac{1-x}{y-4}}$

- Maldak

$P_1(2,5) \longrightarrow y'=\dfrac{1-2}{5-4}=\boxed{-1}$

$P_2(2,3) \longrightarrow y'=\dfrac{1-2}{3-4}=\boxed{1}$

- ukitzaileak

$\boxed{y=y_0+m(x-x_0)}$

- $P_1(2,5) \quad m_1=-1$

$y=5-1(x-2)$

$\boxed{y_1=-x+7}$

- $P_2(2,3) \quad m_2=1$

$y=3-1(x-2)$

$\boxed{y_2=-x+5}$

d) $y=(x^2+1)^{\sin x}$ $\quad x_0=0.$

Zuzen ukitzailea $\quad y=y_0+m(x-x_0)$

- Puntua (x_0, y_0)

$x_0=0 \longrightarrow y=(0^2+1)^{\sin 0}=1^0=1 \longrightarrow \boxed{P(0,1)}$

- Deribatua

$y=(x^2+1)^{\sin x}$ Deribatu logaritm.

$\ln y=\ln(x^2+1)^{\sin x}$

$\ln y=\sin x\cdot\ln(x^2+1)$

$\frac{y'}{y}=\cos x\cdot\ln(x^2+1)+\sin x\,\frac{2x}{x^2+1}$

$y'=y\left[\cos x\cdot\ln(x^2+1)+\frac{2x\cdot\sin x}{x^2+1}\right]$

$y'=(x^2+1)^{\sin x}\cdot\left[\cos x\cdot\ln(x^2+1)+\frac{2x\cdot\sin x}{x^2+1}\right]$

- Malda

$P(0,1)\rightarrow y'=(0^2+1)^{\sin 0}\cdot\left[\cos 0\,\cancel{\ln(1)}^{0}+\cancel{\frac{2\cdot 0\cdot\sin 0}{0^2+1}}^{0}\right]$

$y'=0\longrightarrow\boxed{m=0}$

- Ukitzailea

$P(0,1)$, $m=0$ $\longrightarrow y=1+0(x-0)$

$\boxed{y=1}$

29. orr. $\boxed{2}$ $y=\frac{2x}{x-1}$ ren ukitzaileak $2x+y=0$ rekiko parolel.

- Ukitzaileo eta zuzena PARALELAK badira → MALDA BARDINA

$2x+y=0 \longrightarrow y=-2x \longrightarrow \boxed{m=-2}$

- Malda, deribatua x_0 puntuan da. beraz deribatu kalkulatu

$$f'(x)=\frac{2(x-1)-2x\cdot 1}{(x-1)^2}=\frac{\cancel{2x}-2-\cancel{2x}}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

- Maldarekin berdindut:

$$\frac{-2}{(x-1)^2}=-2 \Longrightarrow -2=-2(x-1)^2$$

$$1=(x-1)^2$$

$$\cancel{1}=x^2-2x+\cancel{1}$$

$$x^2-2x=0 \quad \boxed{x_1=0} \quad \boxed{x_2=2}$$

$$x(x-2)=0$$

$$x_1=0 \longrightarrow f(0)=\frac{2\cdot 0}{0-1}=0 \longrightarrow P_1(0,0)$$

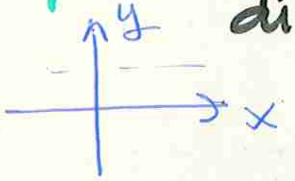
$$x_2=2 \longrightarrow f(2)=\frac{2\cdot 2}{2-1}=4 \longrightarrow P_2(2,4)$$

Ukitzaileak

$$y_1=0-2(x-0) \longrightarrow \boxed{y_1=-2x}$$

$$y_2=4-2(x-2) \longrightarrow \boxed{y_2=-2x+8}$$

29 / $\boxed{3}$ kalkulatu x ardatzarekiko ∥ diren zuzen ukitzaileak.



Berаz ukitzaileen malda $\boxed{m=0}$

beraz $f'(x_0)=0$

a) $y = x \cdot \ln x$

$f'(x) = 1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}$

$m=0 \longrightarrow \ln x + 1 = 0$

$\ln x = -1$

$e^{-1} = x$

$f(e^{-1}) = \frac{\ln e^{-1}}{e} = -\frac{1}{e}$

$\boxed{P\left(\frac{1}{e}, \frac{-1}{e}\right)}$

Ukitzailea $y = y_0 + m(x - x_0)$

$y = -\frac{1}{e} + 0 \cdot (x - e^{-1})$

$\boxed{y = -1/e}$

b) $y = x^2 \cdot e^x$

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x$

$m=0 \quad 2x e^x + x^2 e^x = 0$

$e^x (2x + x^2) = 0$

$e^x \neq 0$

$2x + x^2 = 0$

$x(2+x) = 0$ $\boxed{x_1 = 0}$ $\boxed{x_2 = -2}$

$x_1 = 0$

$f(0) = 0^2 e^0 = 0$

$\boxed{P_1(0,0)}$

$x_2 = -2$

$f(-2) = (-2)^2 \cdot e^{-2} = 4 \cdot e^{-2}$

$\boxed{P_2(-2, 4e^{-2})}$

Ukitzaileak $y = y_0 + m(x - x_0)$

$y_1 = 0 + 0(x - 0) \longrightarrow \boxed{y_1 = 0}$

$y_2 = 4e^{-2} + 0 \cdot (x+2) \longrightarrow \boxed{y_2 = 4 \cdot e^{-2}}$

c) $y = \sin 2x$

$f'(x) = 2 \cdot \cos(2x)$

$m = 0 \longrightarrow 2\cos 2x = 0$

$\cos 2x = 0$

$$\begin{cases} 2x = \frac{\pi}{4} + 2\pi k \longrightarrow x_1 = \frac{\pi}{4} + \pi k \longrightarrow y_1 = 1 \\ 2x = \frac{3\pi}{4} + 2\pi k \longrightarrow x_2 = \frac{3\pi}{4} + \pi k \longrightarrow y_2 = -1 \end{cases}$$

$P_1 \left(\frac{\pi}{4} + \pi k, 1\right) \quad k \in \mathbb{R}$

$P_2 \left(\frac{3\pi}{4} + \pi k, -1\right) \quad k \in \mathbb{R}$

UKITZAILEAK

45

$y_1 = 1 + 0 \cdot \left(x - \frac{\pi}{4} + \pi k\right) \longrightarrow$

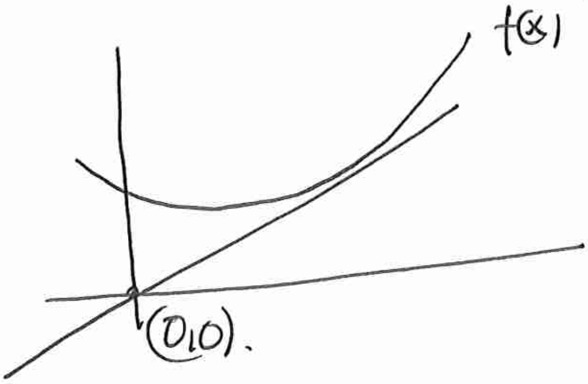
$y_2 = -1 + 0\left(x - \frac{3\pi}{4} + \pi k\right) \longrightarrow$

$\frac{270}{2}$

$$\boxed{\begin{matrix} y_1 = 1 \\ y_2 = -1 \end{matrix}}$$

297/7) $y = 3x^2 - 5x + 12$.

ukitzailea. koordenatu jatorritik. → $(0,0)$.



Zuzen ukitzailea kalkulatzeko $P(0,0)$ eta malda behar dira

$$y = \underset{f(x)}{\underset{\downarrow}{y_0}} + \underset{f'(x_0)}{\underset{\downarrow}{m}}(x - x_0)$$

· Malda kalkulatzeko $\boxed{m = f'(x_0)}$

$f'(x) = 6x - 5$

$f'(0) = -5 \longrightarrow m = -5$

· Beraz ukitzailea:

$P(0,0)$, $m = -5$ $\longrightarrow$ $y = 0 + (-5)\cdot(x - 0)$

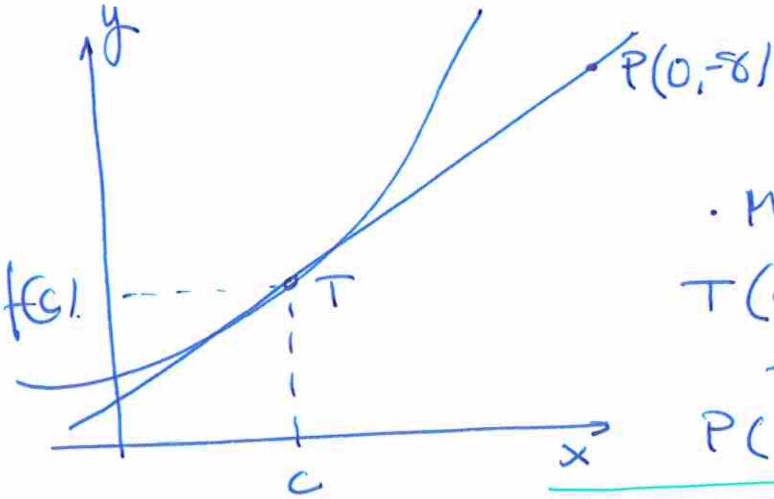
$$\boxed{y = -5x}$$

297/8). $y = \frac{1}{4}x^2 + 4x - 4$.

Zer puntutako ukitzaileak igarotzen ~~dira~~ dira $(0,-8)$tik.

$(0,-8)$ puntua KANPOKO PUNTUA da ~~$f(-8) = \frac{1}{4}(-8)^2 + 4(-8) - 4$~~

$f(0) = \frac{1}{4}\cdot 0^2 + 4\cdot 0 - 4 = -4 \neq -8$ delako.



Zuzen ukitzailea kalkulatzeko $\boxed{m = f'(c)}$

· Malda m_{PT}

$T(c, f(c)) = T(c, \frac{1}{4}c^2 + 4c - 4)$

$f(c) = \frac{1}{4}c^2 + 4c - 4$

$P(0,-8)$

$$\boxed{m = \frac{\frac{1}{4}c^2 + 4c - 4 - (-8)}{c}}$$

- Lehenengo deribatua.

$f'(c) \rightarrow f'(x) = \frac{2}{4}x + 4 \rightarrow f'(c) = \frac{1}{2}c + 4.$

- Planteatuz $m = f'(c)$

$$\frac{\frac{1}{4}c^2 + 4c + 4}{c} = \frac{1}{2}c + 4.$$

$$\frac{1}{4}c^2 + 4c + 4 = c\left(\frac{1}{2}c + 4\right)$$

$$\frac{c^2}{4} + \frac{4c}{} + 4 = \frac{c^2}{2} + 4c$$

$$c^2 + \cancel{16c} + 16 = 2c^2 + \cancel{16c}$$

$$-c^2 = -16$$

$$\boxed{c = \pm 4.}$$

- Bi zuzen ukitzaile:

$c_1 = 4 \rightarrow f(4) = \frac{4^2}{4} + 4 \cdot 4 - 4 = \cancel{4} + 16 - \cancel{4} = 16$

$P(4, 16)$

$m = \frac{1}{2} \cdot 4 + 4 = 6.$

$y_1 = 16 + 6(x - 4) \rightarrow \boxed{y_1 = 6x - 8}$

$c_2 = -4 \rightarrow f(-4) = \frac{1}{4}(-4)^2 + 4(-4) - 4 = \cancel{4} - 16 - \cancel{4} = -16$

$P(-4, -16)$

$m = \frac{1}{2}(-4) + 4 = 2.$

$y_2 = -16 + 2(x - (-4))$

$\rightarrow \boxed{y_2 = 2x - 8}$

29x/10a) $y = x^3 - 6x^2 + 9x$ $\quad Dom f = \mathbb{R}$.

$f'(x) = 3x^2 - 12x + 9$

$f'(x) = 0 \longrightarrow 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$x = \dfrac{4 \pm \sqrt{16-12}}{2} = \dfrac{4 \pm 2}{2} = \begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$

$f'(x)$	$f'>0$	1	$f'<0$	3	$f'>0$
$f(x)$		MAX $(1,4)$		MIN $(3,0)$	

$f(3) = 3^3 - 6 \cdot 3^2 + 9 \cdot 3 = 0.$

$f(1) = 1 - 6 \cdot 1 + 9 = 4.$

KURBADURA

$f''(x) = 6x - 12$

$f''(x) = 0 \longrightarrow 6x - 12 = 0 \longrightarrow \boxed{x = 2}$

$f''(x)$	$f''<0$	2	$f''>0$
$f(x)$	$\cap$		$\cup$

$f'''(x) = 6.$ $\quad f''(2) = 0 \longrightarrow x = 2$ INF PUNTUA

GORAK. TARTEA	$(-\infty, 1) \cup (3, +\infty)$
BEHER TARTEA	$(1, 3)$
MAX ERLAT.	$(1, 4)$
MIN ERLAT.	$(3, 0)$
AHURTASUN TARTEA	$(2, +\infty)$
GANBILTASUN TARTEA	$(-\infty, 2)$

10

c/ $y = x^4 - 2x^3$

$Dom f = \mathbb{R}$ Jarraia eta deribagarria.

$f'(x) = 4x^3 - 6x^2$

$4x^3 - 6x^2 = 0 \rightarrow x^2(4x-6) = 0 < \begin{matrix} x_1 = 0 \\ x_2 = 3/2 \end{matrix}$

		0		1		3/2	
$f(x) = x^4 - 2x^3$	↘		↘			MIN $(3/2, -27/16)$	↗
$f'(x) = 4x^3 - 6x^2$	$f'(-1) < 0$ −	0	$f'(1) < 0$ −				$f'(2) > 0$ +
$f''(x) = 12x^2 - 12x$	$f''(-1) > 0$ + ∪	(0,0) INF PUNT	− ∩	(1,-1) INF P.	+ ∪		

$f''(x) = 12x^2 - 12x$

$12x^2 - 12x = 0 \rightarrow 12x(x-1) = 0 < \begin{matrix} x_1 = 1 \\ x_2 = 0 \end{matrix}$

G.T $(3/2, +\infty)$

B.T $(-\infty, 0) \cup (0, 3/2)$

AHURT $(-\infty, 0) \cup (1, +\infty)$

GANB. $(0, 1)$

MIN $(3/2, -27/16)$

INF. PUNTUA $(0,0)$ eta $(1,-1)$

d/ $y = x^4 + 2x^2$ $Dom f = \mathbb{R}$ Jarraia eta deribagarria.

$f'(x) = 4x^3 + 4x$ $4x(x^2+1) = 0 \rightarrow \boxed{x = 0}$

$f''(x) = 12x^2 + 4$ $4(3x^2+1) = 0$

		0	
$f(x) = x^4 + 2x^2$	↘	MIN $(0,0)$	↗
$f'(x) = 4x^3 + 4x$	$f'(-1) = -8$ −		$f'(1) = 8$ +
$f''(x) = 12x^2 + 4$		+ ∪	

GT $(0, +\infty)$

BT $(-\infty, 0)$

Ah. $(-\infty, +\infty)$

MIN. $(0,0)$.

f/ $y = e^x(x-1)$

Dom f = $\mathbb{R}$ Jarraia eta deribagarria

$f'(x) = e^x\cdot(x-1) + e^x\,(\cancel{x-1+1})$

$f'(x) = e^x \cdot x$

$e^x \cdot x = 0 \begin{cases} e^x \neq 0 \\ \boxed{x=0} \end{cases}$

MIN$(0,-1)$

$f(x) = e^x(x-1)$ — 0

$f'(-1) = -$ $-$ $f'(1)>0$ $+$

$f'(x) = e^x \cdot x$ — -1

$f''(-2)<0$ $-$ $\cap$ INF PUN $(-1,-2e^{-1})$ $f''(0)>1$ $+$ $\cup$

$f''(x) = e^x(x+1)$

$f''(x) = e^x \cdot x + e^x = e^x(x+1) \begin{cases} e^x \neq 0 \\ x = -1 \end{cases}$

$f'''(x) = e^x(x+1) + e^x = e^x(x+2)$

$f'''(-1) = e^{-1}(-1+2) = e^{-1} \neq 0$

GT $(0,+\infty)$
BT $(-\infty,0)$
Ahurr $(-1,+\infty)$
Ganbil $(-\infty,-1)$
MIN $(0,-1)$
Inflexio Punt $\left(-1, \frac{-2}{e}\right)$

297. or) 10f) $y = e^x \cdot (x-1)$

$Dom f = \mathbb{R}$.

HAZKUNDEA: $f'(x) = e^x(x-1) + e^x = e^x \cdot x$

$f'(x) = 0 \rightarrow 0 = \underbrace{e^x}_{\neq 0} \cdot \underbrace{x}_{0}$ $\begin{cases} e^x \neq 0 \\ \boxed{x = 0} \end{cases}$ PN sing.

$f'(x)$	$f' < 0$		$f' > 0$
$f(x)$	$\searrow$	0	$\nearrow$

MIN. $f(0) = e^0(0-1) = -1$

$(0, -1)$

KURBADURA: $f''(x) = e^x \cdot x + e^x = e^x(x+1)$

$f''(x) = 0 \longrightarrow 0 = e^x \cdot (x+1)$ $\begin{cases} e^x \neq 0 \\ x+1 = 0 \rightarrow \boxed{x = -1} \end{cases}$

$f''(x)$	$f'' < 0$		$f'' > 0$
$f(x)$	$\cap$	-1	$\cup$

IP $x = -1$ $f(-1) = e^{-1}(-1-1)$
$= -2/e$

GORAKORRA	$(0, +\infty)$	AHURRA	$(-1, +\infty)$
BEHERAK.	$(-\infty, 0)$	GANBILA	$(-\infty, -1)$
MINIMO	$(0, -1)$	IP	$(-1, -2/e)$

11a) $y = \dfrac{8-3x}{\underbrace{x(x-2)}_{x^2-2x}}$

$Dom f = \mathbb{R} - \{0, 2\}$

$$f'(x) = \frac{-3(x^2-2x) - (8-3x)(2x-2)}{(x^2-2x)^2} = \frac{-3x^2 + \cancel{6x} - (16x - 16 - 6x^2 \cancel{+6x})}{(x^2-2x)^2}$$

$$= \frac{3x^2 - 16x + 16}{(x^2-2x)^2}$$

$f'(x) = 0 \rightarrow 3x^2 - 16x + 16 = 0$

$$x = \frac{16 \pm \sqrt{16^2 - 4 \cdot 3 \cdot 16}}{2 \cdot 3} = \begin{cases} x_1 = 4 \\ x_2 = 4/3 \end{cases}$$

$f'(x)$	$f' > 0$		$f' > 0$		$f' < 0$		$f' < 0$		$f' > 0$
$f(x)$	$\nearrow$	0	$\nearrow$	4/3 MAX.	$\searrow$	2	$\searrow$	4 MIN	$\nearrow$

max $(4/3, 9/2)$

min $(4, -1/2)$

max erl. $x=4/3 \longrightarrow f\left(\frac{4}{3}\right)=\frac{8-\cancel{3}(4/\cancel{3})}{4/3\cdot(4/3-2)}=\frac{4}{-8/9}=9/2$

MIN $\quad x=4 \longrightarrow f(4)=\frac{8-12}{4(4-2)}=\frac{-4}{8}=\frac{-1}{2}.$

KURBADURA

$$f''(x)=\frac{(6x-16)(x^2-2x)^2-2(x^2-2x)(2x-2)(3x^2-16x+16)}{(x^2-2x)^4}$$

$$=\frac{(6x-16)(x^2-2x)-(4x-4)(3x^2-16x+16)}{(x^2-2x)^3}=$$

$$=\frac{6x^3-\cancel{12x^2}-16x^2+32x-[12x^3-64x^2+64x-\cancel{12x^2}+64x-64]}{(\quad)^3}=$$

$$=\frac{-6x^3+48x^2-96x+64}{x^3(x-2)^3}$$

11d)

$$y=\frac{2x^2-3x}{2-x}.$$

$Dom\,f=\mathbb{R}-\{2\}$

AB $x=2$.

HAZKUNDEA

$$f'(x)=\frac{(4x-3)(2-x)-(2x^2-3x)(-1)}{(2-x)^2}=$$

$$=\frac{8x-4x^2-6+3x+2x^2-3x}{(2-x)^2}=\boxed{\frac{-2x^2+8x-6}{(2-x)^2}}$$

$$f'(x)=0 \longrightarrow \frac{-2x^2+8x-6}{(2-x)^2}=0$$

$$x-4x+3=0$$

$$x=\frac{4\pm\sqrt{4^2-4\cdot1\cdot3}}{2}=\begin{cases}x_1=3\\ x_2=1\end{cases}$$

PM SINf.

$f'(x)$	$f'<0$	1	$f'>0$	2	$f'>0$	3	$f'<0$
$f(x)$	↘	MIN	↗	AB	↗	MAX	↘

min $f(1)=-1$

max $f(3)=-9$.

KURBADURA

$$f''(x)=\frac{(-4x+8)(2-x)^2-(-2x^2+8x-6)2(2-x)(-1)}{(2-x)^4}=$$

$$=\frac{\cancel{(2-x)}\left[(-4x+8)(2-x)+(-2x^2+8x-6)\cdot 2\right]}{(2-x)^{\cancel{4}\,3}}=$$

$$=\frac{\cancel{-8x}+\cancel{4x^2}+16\cancel{-8x}\cancel{-4x^2}+\cancel{16x}+12}{(2-x)^3}=\frac{4}{(2-x)^3}$$

$$f''(x)=0 \longrightarrow \frac{4}{(2-x)^3}\neq 0.$$

$f''(x)$	$f''>0$	2	$f''<0$
$f(x)$	∪	AB	∩

$$\boxed{\begin{array}{l} G.T\ (1,2)\cup(2,3)\\ BT\ (-\infty,1)\cup(3,+\infty)\\ MAX\ ERL\ (3,-9)\\ MIN\ ERL\ (1,-1)\\ AHURRA\ (-\infty,2)\\ GANBILA\ (2,+\infty)\end{array}}$$

297/ 11b $y = \dfrac{x^2+1}{x^2-1}$ $\quad$ Dom f $= \mathbb{R} - \{\pm 1\}$

$$f'(x) = \frac{2x(x^2-1) - (x^2+1)\cdot 2x}{(x^2-1)^2} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$f'(x) = 0 \longrightarrow \frac{-4x}{(x^2-1)^2} = 0 \longrightarrow \boxed{x=0} \quad \text{Pto sing.}$$

$f'(x)$	$+$		$+$		$-$		$-$
$f(x)$	$\nearrow$	-1	$\nearrow$	0 MAX.	$\searrow$	1	$\searrow$

$x = 0 \longrightarrow f(0) = \dfrac{0^2+1}{0^2-1} = -1$

MAX. $(0, -1)$.

$$f''(x) = \frac{-4(x^2-1)^2 - (-4x)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} =$$

$$= \frac{-4(x^2-1)^2 + 16x^2(x^2-1)}{(x^2-1)^4} = \frac{\cancel{(x^2-1)}\left[-4(x^2-1) + 16x^2\right]}{(x^2-1)^{\cancel{4}\,3}} =$$

$$= \frac{-4x^2+4+16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$

$$f''(x) = 0 \quad \frac{12x^2+4}{(x^2-1)^3} = 0 \quad 12x^2+4 = 0 \longrightarrow x^2 = \frac{-4}{12} \ \nexists$$

$f''(x)$	$f''>0$ $+$		$f''<0$ $-$		$f''>0$ $+$
$f(x)$	$\cup$	-1 AB	$\cap$	1 AB	$\cup$

G.T $(-\infty, -1) \cup (-1, 0)$

BT $(0, 1) \cup (1, +\infty)$

MAX $(0, -1)$

AHORNA $(-\infty, -1) \cup (1, +\infty)$

SANB. $(-1, 1)$

Ahurt., ganbilt. eta inflex.puntuak

12 b) $y = x^4 - 6x^2$

Def. Domf $= \mathbb{R}$.

$f'(x) = 4x^3 - 12x \rightarrow 4x^3 - 12x = 0 \rightarrow 4x(x^2-3) = 0 \begin{cases} x = 0 \\ x = \sqrt{3} \\ x = -\sqrt{3} \end{cases}$

$f''(x) = 12x^2 - 12 \rightarrow 12x^2 - 12 = 0 \rightarrow 12(x^2-1) = 0$

$\rightarrow x_1 = 1$

$x_2 = -1$

$f'(x)$	$-$	$-\sqrt{3}$	$+$	0	$-$	$\sqrt{3}$	$+$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

$f''(x)$	$f''>0$	-1	$f''<0$	1	$f''>0$
$f(x)$	$\cup$	INF PUNT $(-1,-5)$	$\cap$	INF PUNT $(1,-5)$	$\cup$

d) $y = x \cdot e^x$ Domf $= \mathbb{R}$

$\rightarrow f'(x) = e^x + x \cdot e^x = e^x(1+x)$

$f'(x) = 0 \quad 0 = e^x(1+x) < \begin{matrix} e^x \neq 0 \\ \boxed{x = -1} \end{matrix}$ Pu sing.

$\rightarrow f''(x) = e^x(1+x) + e^x = e^x(2+x)$

$f''(x) = 0 \quad 0 = e^x(2+x) < \begin{matrix} e^x \neq 0 \\ \boxed{x = -2} \end{matrix}$

$f'(x)$	$f'<0$	-1	$f'>0$
$f(x)$	$\searrow$	MIN $(-1, -\frac{1}{e})$	$\nearrow$

$f''(x)$	$f''<0$	-2	$f''>0$
$f(x)$	$\cap$	$(-2, \frac{-2}{e^2})$	$\cup$

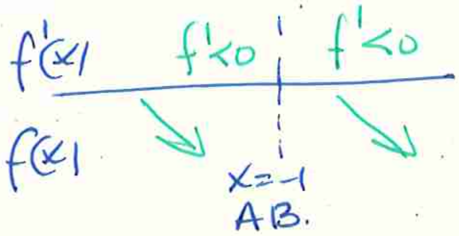
GT $(-1, +\infty)$
BT $(-\infty, -1)$
AHURRA $(-2, +\infty)$
GANBILA $(-\infty, -2)$
INF PUNTUA $(-2, \frac{-2}{e^2})$
MIN $(-1, -1/e)$

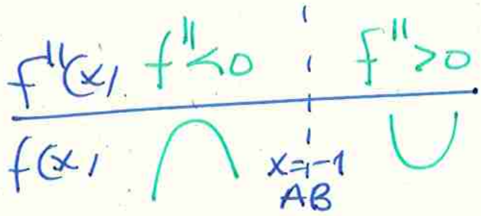
12 e) $y = \dfrac{2-x}{x+1}$ Dom $f = \mathbb{R} - \{-1\}$. AB $x=-1$

$$f'(x) = \frac{-1(x+1)-(2-x)}{(x+1)^2} = \frac{\cancel{-x}-1-2\cancel{+x}}{(x+1)^2} = \frac{-3}{(x+1)^2} = -3(x+1)^{-2}$$

$\nexists\, x_0 / f'(x_0)=0 \longrightarrow$ Ez dago puntu singularrik.

$$f''(x) = \frac{6}{(x+1)^3} \qquad f''(x) \neq 0 \quad x \text{ puntu kritiko}.$$





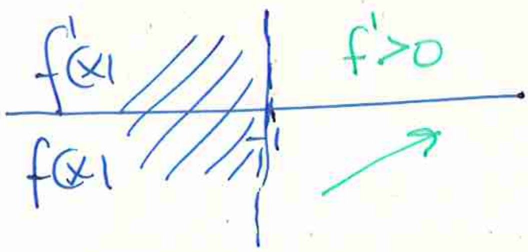
$$\boxed{\begin{array}{l} \text{BT } (-\infty,-1) \cup (-1,+\infty) \\ \text{GT } \text{——} \\ \text{AHURT. } (-1,+\infty) \\ \text{GANBILT. } (-\infty,-1) \end{array}}$$

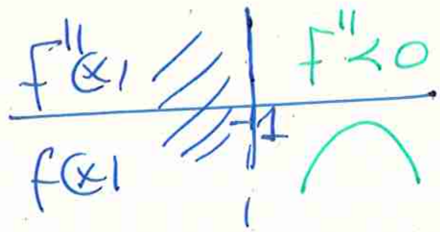
12 f) $y = \ln(x+1)$ Dom $f = (-1,+\infty)$

AB $x=-1$

$$f'(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$f''(x) = \frac{-1}{(x+1)^2}$$





GT $(-1,+\infty)$

GANBILA $(-1,+\infty)$

17) $f(x)=1+\frac{a}{x}+\frac{6}{x^2}$

$x=3$ MUTUR ERLATIBOA $\longrightarrow$ $\boxed{f'(3)=0.}$

$$f'(x)=-\frac{a}{x^2}-\frac{12}{x^3}.$$

$$f'(3)=0 \longrightarrow -\frac{a}{9}-\frac{12}{27}=0$$

$$-3a-12=0 \longrightarrow \boxed{a=-4.}$$

Jakiteko maximo ala minimoa dan, bigarren deribatua erabiltzen da.

$$f''(x)=\frac{-8}{x^3}+\frac{36}{x^4}.$$

$$f''(3)=\frac{-8}{3^3}+\frac{36}{3^4}=\frac{4}{27}>0 \quad \cup \Rightarrow \text{MINIMOA}$$

Puntua $(3, f(3))=$ $\boxed{(3,\frac{1}{3}) \text{ MINIMOA DA}}$

18) $f(x)=ax^3+bx.$

1) (1,1) puntutik pasatzen danez:

$f(1)=1 \longrightarrow$ $\boxed{a\cdot 1^3+b\cdot 1=1}$

2.) (1,1) puntuan $3x+y=0$ zuzenarekiko paraleloa da ukitzailea.

$3x+y=0 \longrightarrow y=-3x \longrightarrow m=-3.$

Maldа -3 bada $\longrightarrow$ $f'(1)=-3.$

$f'(x)=3ax^2+b$

$f'(1)=-3 \longrightarrow$ $\boxed{3a\cdot 1^2+b=-3.}$

Sistema ebatzi:

$$-\begin{cases} a+b=1 \\ 3a+b=-3 \end{cases}$$

$$-2a=4$$

$\boxed{a=-2}$ $\quad b=1-(-2) \rightarrow \boxed{b=3}$

$\boxed{19}$ $f(x)=x^3+ax^2+bx+c.$

1) $x=2$ puntuan MUTUR ERLATIBOA

Mutur erlatiboa izateko $f'(x_0)=0$, beraz:

$f'(x)=3x^2+2ax+b.$

$f'(2)=0 \longrightarrow 3\cdot 2^2+2a\cdot 2+b=0.$

$\boxed{12+4a+b=0.}$

2) $P(1,2)$ puntuan INFLEXIO PUNTUA

Inflexio puntua izateko: $f''(x_0)=0$. beraz.

$f''(x)=6x+2a$

$f'(1)=0 \longrightarrow 6\cdot 1+2a=0 \longrightarrow \boxed{a=-3}$

Berat:

$a=-3 \rightarrow 12+4a+b=0$

$12+4\cdot(-3)+b=0$

$\boxed{b=0.}$

c) c kalkulatzeko: $P(1,2)$ puntuarekin:

$f(1)=2.$

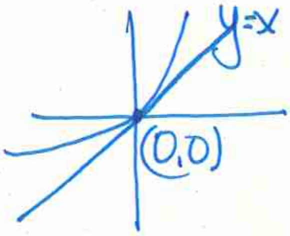
$f(x)=x^3-3x^2+c$

$f(1)=1^3-3\cdot+c=2$

$\boxed{c=4}$

20) $f(x) = x^4 + ax^3 + bx^2 + cx$

a) $x=0$ puntuan ukitzailea $y=x$ da.



Adierazpen honekin $(0,0)$ puntua kurban dago $\rightarrow$ $\boxed{f(0)=0}$

Bestaldetik ukitzailea $y=x$ bada malda 1 da $\rightarrow$ $\boxed{f'(0)=1}$

b) Mutur erlatibo bat du $(-1,0)$ puntuan.

Mutur erlatiboa izateko $f'(x)=0$, beraz $x=-1$ denean $\boxed{f'(-1)=0}$ izango da. eta $(-1,0)$ kurbaren puntu bat denez $\boxed{f(-1)=0}$ da.

4 baldintzak planteatuz $f(x) = x^4 + ax^3 + bx^2 + cx$

$$f(0)=0 \longrightarrow \cancel{0^4} + \cancel{a0^3} + \cancel{b\cdot 0^2} + \cancel{c\cdot 0} = 0$$

$$f(-1)=0 \longrightarrow (-1)^4 + a(-1)^3 + b(-1)^2 + c(-1) = 0.$$

$$f'(x) = 4x^3 + 3ax^2 + 2bx + c$$

$$f'(0) = 1 \longrightarrow 4\cdot 0^3 + \cancel{3a\cdot 0^2} + \cancel{2b(0)} + c = 1 \longrightarrow \boxed{c=1}$$

$$f'(-1)=0 \longrightarrow 4\cdot(-1)^3 + 3a(-1)^2 + 2b(-1) + c = 0$$

Beraz:

$$\begin{cases} 1 - a + b - c = 0 \\ c = 1 \\ -4 + 3a - 2b + c = 0 \end{cases} \qquad \begin{rcases} -a + b = 0 \\ 3a - 2b = 3 \end{rcases}$$

$$\boxed{\begin{aligned} a &= 3. \\ b &= 3 \end{aligned}}$$

21) $f(x) = ax^3 + bx^2 + cx + d$

max erlatiboa $(0,4)$

minimo erlatiboa $(2,0)$

Max. $(0,4)$ izateagatik $\boxed{f(0)=4}$ da eta $\boxed{f'(0)=0}$.

Minimoa $(2,0)$ izateagatik $\boxed{f(2)=0}$ da eta $\boxed{f'(2)=0}$.

$$f'(x) = 3ax^2 + 2bx + c$$

$$\left.\begin{array}{ll} f(0)=4 \rightarrow & 4 = \cancel{a\cdot 0^3} + \cancel{b\cdot 0^2} + \cancel{c\cdot 0} + d \\ f'(0)=0 \rightarrow & 0 = \cancel{3a0^2} + \cancel{2b\cdot 0} + c \\ f(2)=0 \rightarrow & 0 = a\cdot 2^3 + b\cdot 2^2 + c\cdot 2 + d. \\ f'(2)=0 \rightarrow & 0 = 3a\cdot 2^2 + 2b\cdot 2 + c \end{array}\right\}$$

$$\left\{\begin{array}{l} d = 4. \\ c = 0 \\ 8a + 4b + \cancel{2c} + d = 0 \\ 12a + 4b + \cancel{c} = 0 \end{array}\right. \qquad \left.\begin{array}{l} 8a + 4b = -4 \\ 12a + 4b = 0 \end{array}\right\}$$

$$\boxed{\begin{array}{l} a = 1 \\ b = -3 \end{array}}$$

22) $f(x)=x^4+ax^2+bx$

$g(x)=x-cx^2$

I) (1,0) puntutik.

$\boxed{f(1)=0}$ $\boxed{g(1)=0}$

II) (1,0) puntuan zuzen ukitzaile berdina.

Zuzen ukitzailearen malda berdina izango da.

berat $m=f'(x_0)$ danez $\begin{array}{l} f'(1)=m_1 \\ g'(1)=m_2 \end{array} \Rightarrow \boxed{\begin{array}{c} m_1=m_2 \\ f'(1)=g'(1) \end{array}}$

$$\left\{\begin{array}{l} f(x)=x^4+ax^2+bx \longrightarrow f'(x)=4x^3+2ax+b \\ g(x)=x-cx^2 \longrightarrow g'(x)=1-2cx. \end{array}\right.$$

Berat:

$$\left.\begin{array}{l} f(1)=0 \longrightarrow 1^4+a\cdot 1^2+b\cdot 1=0 \longrightarrow a+b=-1 \\ g(1)=0 \longrightarrow 1-c\cdot 1^2=0 \longrightarrow \boxed{c=1} \\ f'(1)=g'(1) \rightarrow 4\cdot 1^3+2\cdot a\cdot 1+b=1-\underbrace{2c}_{-2} \\ \qquad 2a+b=-5 \end{array}\right\}$$

$$\left.\begin{array}{l} a+b=-1 \\ 2a+b=-5 \end{array}\right\}$$

$-a=4 \longrightarrow \boxed{a=-4}$ $\boxed{b=3}$ $\boxed{c=1}$

23| $y = ax^4 + 3bx^3 - 3x^2 - ax$

Inf. puntuak $x=1$ eta $x=1/2$ puntuetan.

Inflexio puntuetan $f''(x)=0$ berat $\left\{ \begin{array}{l} \boxed{f''(1)=0} \\ \boxed{f''(\frac{1}{2})=0.} \end{array} \right.$

$f'(x) = 4ax^3 + 9bx^2 - 6x - a$

$f''(x) = 12ax^2 + 18bx - 6$

$$\left. \begin{array}{l} f''(1)=0 \longrightarrow 12 \cdot a \cdot 1^2 + 18 \cdot b \cdot 1 - 6 = 0. \\ f''(\frac{1}{2})=0 \longrightarrow 12 \cdot a (\frac{1}{2})^2 + 18b \cdot \frac{1}{2} - 6 = 0 \end{array} \right\}.$$

$$\left| \begin{array}{l} 12a + 18b = 6 \\ 3a + 9b = 6 \end{array} \right. \Longrightarrow \boxed{a=-1} \quad \boxed{b=1}$$

$$\left\{ \begin{array}{l} 12a + 18b = 6 \\ -6a - 18b = -12 \end{array} \right.$$

$6a = -6$

24) $y = x^3 + ax^2 + bx + c$

Abzisa-ardatza ebaki $x = -1 \longrightarrow \boxed{f(-1) = 0}$ $(-1, 0)$

Inflexio puntua $(2,1)$, $(2,1)$ puntuak funtzioa

betetzen dau $\longrightarrow \boxed{f(2) = 1}$ eta inflexio puntua

izateagatik $\boxed{f''(2) = 0}$

$y = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$$\left.\begin{array}{ll} f(-1) = 0 \longrightarrow & (-1)^3 + a(-1)^2 + b(-1) + c = 0 \\ f(2) = 1 \longrightarrow & 2^3 + a \cdot 2^2 + b \cdot 2 + c = 1 \\ f''(2) = 0 \longrightarrow & 6 \cdot 2 + 2a = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} -1 + a - b + c = 0 \\ 8 + 4a + 2b + c = 1 \\ 12 + 2a = 0 \longrightarrow \boxed{a = -6} \end{array}\right. \Rightarrow \left.\begin{array}{r} -b + c = 7 \\ 2b + c = 17 \end{array}\right\}$$

$$-3b = -10$$

$$\boxed{b = 10/3}$$

$$\boxed{c = 31/3}$$

25/ $f(x)=x^3+ax^2+bx+c$

- $f(1)=1$ → $P(1,1)$ puntua kurban dago
- $f'(1)=0$ → zuzen ukitzailearen malda $x=1$ denean $m=0$ da. $x=1$ denean PUNTU SINGULARRA dago. (max, min edo IP)
- **f-k ez dauka mutur erlatiborik $x=1$ puntuan**

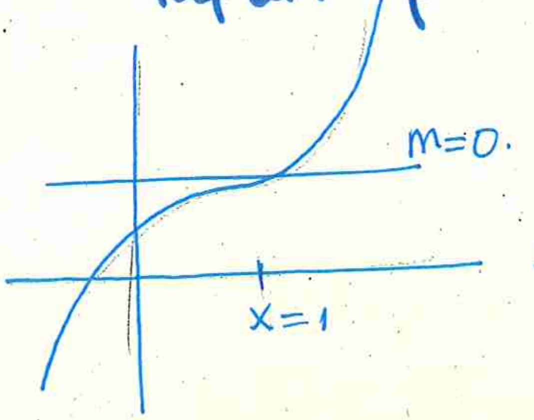
aurreko baldintzarekin $f'(1)=0$, $x=1$, puntu singularra da, eta ez bada mutur erlatiboa, INFLEXIO PTUA izango da. beraz → $\boxed{f''(1)=0}$

$$\left|\begin{array}{l} f(x)=x^3+ax^2+bx+c \\ f'(x)=3x^2+2ax+b. \\ f''(x)=6x+2a. \end{array}\right.$$

$$\left.\begin{array}{l} f(1)=1 \Longrightarrow 1+a+b+c=1 \\ f'(1)=0 \Longrightarrow 3+2a+b=0 \\ f''(1)=0 \Longrightarrow 6+2a=0 \end{array}\right\} \quad \begin{array}{l} \boxed{a=-3} \\ \boxed{b=3} \\ \boxed{c=0} \end{array}$$

26) $f(x) = x^3 + ax^2 + bx + 5$

$x = 1$ puntuan ukitzaile horizontaleko inflexio-puntua.



* Zuzen ukitzaile horizontala bada $m = 0 \longrightarrow \boxed{f'(1) = 0}$

* Eta inflexio puntua bada $\boxed{f''(1) = 0.}$

$$\left.\begin{array}{lll} f(x) = x^3 + ax^2 + bx + 5 & & \\ f'(x) = 3x^2 + 2ax + b & \rightarrow f'(1) = 0 & 3 \cdot 1^2 + 2a \cdot 1 + b = 0 \\ f''(x) = 6x + 2a & \longrightarrow f''(1) = 0 & 6 \cdot 1 + 2a = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} 3 + 2a + b = 0 \rightarrow \\ 6 + 2a = 0 \longrightarrow \end{array}\right. \boxed{\begin{array}{l} b = 3 \\ a = -3 \end{array}}$$

27) $y = \dfrac{e^x}{x^2+c}$

Puntu kritiko bakarra. (max, min edo Inf. ptu).

Ptu kritikoa izateko $f'(x) = 0$.

$$y' = \frac{e^x(x^2+c) - 2x \cdot e^x}{x^2+c} = \frac{e^x(x^2-2x+c)}{x^2+c}$$

$$y' = 0 \longrightarrow e^x(x^2-2x+c) = 0 \begin{cases} e^x \neq 0 \\ x^2-2x+c = 0 \end{cases}$$

Ptu bakarra izateko:

$$x = \frac{2 \pm \sqrt{4-4 \cdot c}}{2} \qquad 4-4c = 0 \qquad \boxed{c = 1.}$$

Beraz funtzioa eta deribatuak horrela dira:

$$y = \frac{e^x}{x^2+1}$$

$$y' = \frac{e^x(x^2-2x+1)}{x^2+1}$$

Ptu kritikoan $y' = 0 \longrightarrow x^2-2x+1 = 0$

$$x = \frac{2 \pm \sqrt{4-4}}{2} = 1.$$

Ptua $(1, f(1)) = \left(1, \frac{e}{2}\right)$.

		1	
$f'(x)$	$f'>0$		$f'>0$
$f(x)$	$\nearrow$		$\nearrow$

Inflexio puntua da