322) 136) $y = \dfrac{x^2-2x+2}{x-1}$

1) DEFINIZIO EREMUA $\boxed{\mathrm{Dom}\, f = \mathbb{R} - \{1\}}$

2) EBAKETA PUNTUAK

OY ARDATZA $x=0$ $y = \dfrac{0^2-2\cdot 0+2}{0-1} = -2$ $\boxed{P(0,-2)}$

OX ARDATZA $y=0$ $0 = \dfrac{x^2-2x+2}{x-1}$ $x = \dfrac{2 \pm \sqrt{2^2-4\cdot 2}}{2} \nexists x$

3) SIMETRIA

$f(-x) = \dfrac{(-x)^2-2(-x)+2}{(-x)-1} = \dfrac{x^2+2x+2}{-x-1}$ EZ DAUKA SIMETRIARIK

4) Ez da periodikoa

5) ASINTOTAK

$\boxed{AB}$ $\lim\limits_{x\to D} f(x) = \pm\infty$. $x-1 \neq 0 \longrightarrow$ $\boxed{x=1 \text{ A. BERTIKALA}}$

0,999 → ← 1,001

$\lim\limits_{x\to 1^-} \dfrac{x^2-2x+2}{x-1} = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{x\to 1^+} \dfrac{x^2-2x+2}{x-1} = \dfrac{1}{0^+} = +\infty$

$\boxed{AZ}$ dago P(x) ren maila > Q(x) mail bat ($\deg P(x) = \deg Q(x)+1$)

$$\begin{array}{r|l} x^2-2x+2 & x-1 \\ \hline -x^2+x & x-1 \\ -x+2 & \\ +x-1 & \\ \hline 1 & \end{array}$$

$f(x) = \underbrace{x-1}_{AZ} + \underbrace{\dfrac{1}{x-1}}_{\text{distantzia}}$ AZ $\boxed{y=x-1}$

$f(x) - (x-1) = \dfrac{1}{x-1}$

$x\to+\infty$ $\dfrac{1}{x-1} = \oplus$ funtzioa asintotaren GAINETIK

$x\to-\infty$ $\dfrac{1}{x-1} = \ominus$ funtzioa asintotaren AZPITIK

Beste modu batera:

$m = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x^2-2x+2}{x^2-x} = \boxed{1}$

$n = \lim\limits_{x\to\infty} (f(x) - mx) = \lim\limits_{x\to\infty} \left(\dfrac{x^2-2x+2}{x-1} - x\right) =$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}-2x+2-\cancel{x^2}+x}{x-1} = \lim\limits_{x\to\infty} \dfrac{-x+2}{x-1} = \boxed{-1}$

6,7) HAZKUNDEA ETA MUTUR ERLATIBOAK

$$f(x)=\frac{x^2-2x+2}{x-1}$$

$$f'(x)=\frac{(2x-2)(x-1)-(x^2-2x+2)}{(x-1)^2}=\frac{2x^2-\cancel{2x}-2x+\cancel{2}-x^2+\cancel{2x}-\cancel{2}}{(x-1)^2}$$

$$f'(x)=\frac{x^2-2x}{(x-1)^2}\qquad f'(x)=0\longrightarrow 0=\frac{x^2-2x}{(x-1)^2}\qquad \boxed{\begin{matrix}x_1=0\\ x_2=2\end{matrix}}$$

				AB			
$f'(x)$	$f'>0$	0	$f'<0$	1	$f'<0$	2	$f'>0$
$f(x)$	$\nearrow$	MAX $(0,-2)$	$\searrow$		$\searrow$	MIN $(2,2)$	$\nearrow$

GT $(-\infty,0)\cup(2,+\infty)$
BT $(0,1)\cup(1,2)$
Max $(0,-2)$
Min $(2,2)$

8) AHURT/GANBILT.

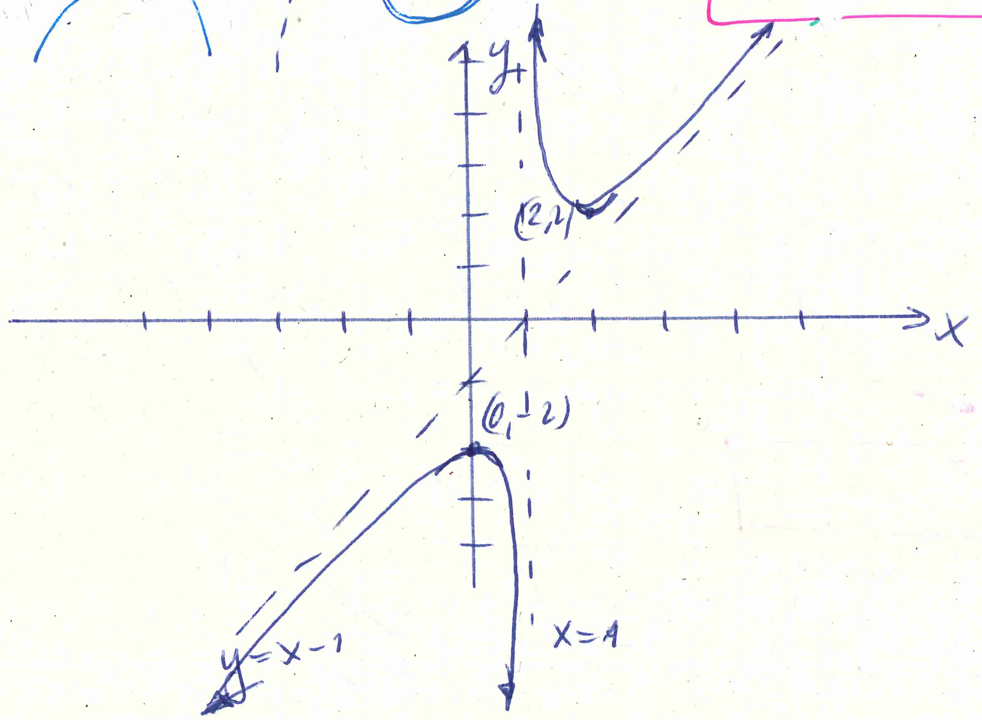
$$f''(x)=\frac{(2x-2)(x-1)^{\cancel{2}}-2\cancel{(x-1)}(x^2-2x)}{(x-1)^{\cancel{4}3}}$$

$$f''(x)=\frac{(2x-2)(x-1)-2(x^2-2x)}{(x-1)^3}=\frac{\cancel{2x^2}-\cancel{2x}-\cancel{2x}+2-\cancel{2x^2}+\cancel{4x}}{(\quad)^3}$$

$$f''(x)=\frac{2}{(x-1)^3}\qquad \nexists x/f''(x)=0.$$ Ez dago inflexio-puntuik.

		AB	
$f''(x)$	$f''<0$	1	$f''>0$
$f(x)$			

AHURRA $(1,+\infty)$
GANBILA $(-\infty,1)$



ADIERAZPEN GRAFIKOA

322) 13a) $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}.$

1) DEFINIZIO EREMUA

$x^2 + x + 1 \neq 0 \qquad x = \dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 1}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2} \quad \nexists x.$

$\boxed{Dom f = \mathbb{R}.}$

2) EBAKETA PUNTUAK

OX ARDATZA $y = 0 \longrightarrow$ EZ DAGO EBAKETARIK

$0 = \dfrac{x^2 - x + 1}{x^2 + x + 1} \longrightarrow x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 4}}{2} \quad \nexists x$

OY ARDATZA $x = 0$

$f(0) = \dfrac{0^2 - 0 + 1}{0^2 + 0 + 1} = 1 \longrightarrow \boxed{(0, 1)}$

3) SIMETRIA

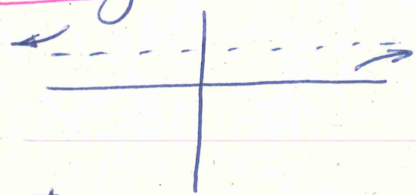
$f(-x) = \dfrac{(-x)^2 - (-x) + 1}{(-x)^2 - x + 1} = \dfrac{x^2 + x + 1}{x^2 - x + 1}$ EZ DA BAKOITIA eta EZ DA BIKOITIA $\rightarrow$ EZ DAUKA SIMETRIARIK

4) Ez da PERIODIKOA

5) ASINTOTAK

AB $\lim_{x \to ?} f(x) = \infty \qquad x^2 + x + 1 \neq 0 \rightarrow$ Ez dauka AB

AH $\lim_{x \to +\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} = \left(\dfrac{+\infty}{+\infty}\right) = 1 \longrightarrow \boxed{AH \Rightarrow y = 1}$



$\lim_{x \to -\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} \left(\dfrac{+\infty}{+\infty}\right) = 1$

$\lim_{x \to +\infty} f(x) = 1^-$ azpitik $\qquad \lim_{x \to -\infty} f(x) = 1^+$ gainetik

Beste modu

$f(x) - A = \dfrac{x^2 - x + 1}{x^2 + x + 1} - 1 = \dfrac{\cancel{x^2} - x + \cancel{1} - (\cancel{x^2} + x + \cancel{1})}{x^2 + x + 1} = \dfrac{-2x}{x^2 + x + 1}.$

$\dfrac{-2x}{x^2 + x + 1}$
- $x \to +\infty \qquad \dfrac{-}{+} = \ominus$ azpitik
- $x \to -\infty \qquad \dfrac{+}{+} = \oplus$ gainetik

6/7 HAZKUNDEA ETA MUTUR ERLATIBOAK

$$f'(x) = \frac{(2x-1)(x^2+x+1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2} =$$

$$= \frac{\cancel{2x^3} + \cancel{2x^2} + \cancel{2x} - \cancel{x^2} - \cancel{x} - 1 - \cancel{2x^3} - \cancel{x^2} + 2x^2 + \cancel{x} - \cancel{2x} - 1}{(\quad)^2} =$$

$$\boxed{f'(x) = \frac{2x^2-2}{(x^2+x+1)^2}}$$

$$f'(x)=0 \quad \frac{2x^2-2}{(\quad)^2} = 0 \rightarrow 2x^2-2=0$$

$$x = \pm 1.$$

$f'(x)$	$f'>0$	-1	$f'<0$	1	$f'>0$
$f(x)$	↗	MAX $(-1,3)$	↘	MIN $(1,1/3)$	↗

$f(-1) = 3 \quad (-1,3)$

$f(1) = 1/3 \quad (1, 1/3)$

GT $(-\infty,-1) \cup (1,+\infty)$
BT $(-1,1)$

8/ AHURTASUNA ETA GANBILTASUNA

$$f''(x) = \frac{4x(x^2+x+1)^{\cancel{2}} - (2x^2-2)2\cancel{(x^2+x+1)}\cdot(2x+1)}{(x^2+x+1)^{\cancel{4}\,3}} =$$

$$= \frac{4x(x^2+x+1) - 2(2x^2-2)(2x+1)}{(x^2+x+1)^3} =$$

$$= \frac{4x^3 + \cancel{4x^2} + 4x - 8x^3 - \cancel{4x^2} + 8x + 4}{(\quad)^3} = \boxed{\frac{-4x^3+12x+4}{(x^2+x+1)^3} = f''(x)}$$

$$f''(x)=0 \quad \frac{-4x^3+12x+4}{(x^2+x+1)^3} = 0 \quad -4x^3+12x+4=0$$

Ruffinirekin ezin da

Ezin da kalkulatu IP.

$f''(x)$	-1	1
$f(x)$