# 11g) $y=\dfrac{x^3}{1-x^2}$

1.) DEFINIZIO EREMUA $\quad \boxed{Dom f = \mathbb{R}-\{-1,1\}}$

$1-x^2 \neq 0 \quad x=\pm 1.$

2) EBAKETA PUNTUAK

OY ardatza $\;x=0 \quad y=0 \longrightarrow \boxed{P(0,0)}$

OX ardatza $\;y=0 \longrightarrow x=0$

3/ SIMETRIA

$$\boxed{f(-x)}=\frac{(-x)^3}{1-(-x)^2}=\frac{-x^3}{1-x^2}=\boxed{-f(x)}$$

SIMETRIA BAKOITIA

$f(-x)=-f(x)$

4/ PERIODIKOTASUNA $\rightarrow$ Ez da periodikoa.

5/ ASINTOTAK.

$\boxed{\text{AB.}\quad x=1,\ x=-1}$

$x=1.\quad \lim\limits_{x\to 1}\dfrac{x^3}{1-x^2}=\left(\dfrac{1}{0}\right)$

$\lim\limits_{\substack{x\to 1^-\\ 0{,}99}}\dfrac{x^3}{1-x^2}=\dfrac{1}{0^+}=+\infty$

$\lim\limits_{\substack{x\to 1^+\\ 1{,}001}}\dfrac{x^3}{1-x^2}=\dfrac{1}{0^-}=-\infty$

$x=-1\quad \lim\limits_{x\to -1}\dfrac{x^3}{1-x^2}=\left(\dfrac{1}{0}\right)$

$\lim\limits_{\substack{x\to -1^-\\ -1{,}001}}\dfrac{x^3}{1-x^2}=\dfrac{-1}{0^-}=+\infty$

$\lim\limits_{\substack{x\to -1^+\\ -0{,}999}}\dfrac{x^3}{1-x^2}=\dfrac{-1}{0^+}=-\infty.$

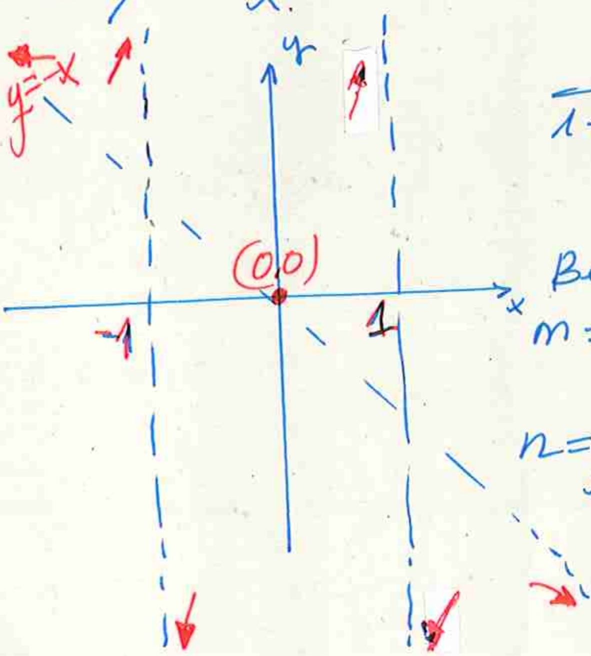
A. ZEIHARRA

$$\begin{array}{ll|l} x^3 & & -x^2+1 \\ \hline -x^3 & +x & -x \\ \hline / & x & \end{array}$$

$$\frac{x^3}{1-x^2}=\underbrace{-x}_{\text{A. zeiharra}}+\underbrace{\frac{x}{1-x^2}}_{\text{kurbatik asintotarekiko distantzia}} \qquad \boxed{y=-x}\ \text{A.Z.}$$

$\dfrac{x}{1-x^2}$

- $x\to+\infty \quad \dfrac{+}{-}=\ominus$ kurba asintotaren AZPITIK
- $x\to-\infty \quad \dfrac{-}{-}=\oplus$ kurba GAINETIK.




Beste moduna

$$m=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^3}{(1-x^2)x}=\boxed{-1=m}$$

$$n=\lim_{x\to\infty} f(x)-mx=\lim_{x\to\infty}\frac{x^3}{1-x^2}-(-1)x=$$

$$=\lim_{x\to\infty}\frac{x^3+x(1-x^2)}{1-x^2}=\lim_{x\to\infty}\frac{x}{1-x^2}=\boxed{0=n}$$

$\boxed{y=-x}$

## 6/ HAZKUNDEA

$$f(x) = \frac{x^3}{1-x^2} \qquad f'(x) = \frac{3x^2(1-x^2) - x^3(-2x)}{(1-x^2)^2} =$$

$$= \frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{-x^4 + 3x^2}{(1-x^2)^2}$$

Kontutan izan
$$< \begin{array}{l} f'(x)=0 \\ \nexists f'(x) \longrightarrow x = \pm 1 \end{array}$$

$$f'(x)=0 \longrightarrow -\frac{x^4+3x^2}{(1-x^2)^2} = 0 \qquad x^2(-x^2+3) = 0$$

$x_1 = 0$, $x_2 = +\sqrt{3}$, $x_3 = -\sqrt{3}$ } Ptu supul.

| | $-\sqrt{3}$ | | $-1$ | $0$ | | $1$ | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|---|
| $f'(x)$ | $f'<0$ | $f'>0$ | | $f'>0$ | $f'>0$ | | $f'>0$ | $f'<0$ |
| $f(x)$ | $\searrow$ MIN | $\nearrow$ | AB | $\nearrow$ | $\nearrow$ | AB | $\nearrow$ MAX | $\searrow$ |

MIN: $(-\sqrt{3}, +2{,}6)$ — $(-1{,}73; +2{,}6)$

MAX: $\left(\sqrt{3}, -\frac{3\sqrt{3}}{2}\right)$ — $(1{,}73; -2{,}6)$

## 7/ AHURT./GANBILTASUNA

$$f''(x) = \frac{(-4x^3+6x)(1-x^2)^2 - (-x^4+3x^2)\,2(1-x^2)(-2x)}{(1-x^2)^4}$$

$$= \frac{\cancel{(1-x^2)}\left[(-4x^3+6x)(1-x^2) + 4x(-x^4+3x^2)\right]}{(1-x^2)^{\cancel{4}\,3}} =$$

$$= \frac{-4x^3 + \cancel{4x^5} + 6x - 6x^3 - \cancel{4x^5} + 12x^3}{(1-x^2)^3} = \frac{2x^3+6x}{(1-x^2)^3}$$

$$f''(x)=0 \longrightarrow \frac{2x^3+6x}{(1-x^2)^2} = 0 \qquad \begin{array}{l} 2x^3+6x=0 \\ 2x(x^2+3)=0 \end{array} < \begin{array}{l} \boxed{x=0} \\ \cancel{x=\pm\sqrt{-3}} \end{array}$$

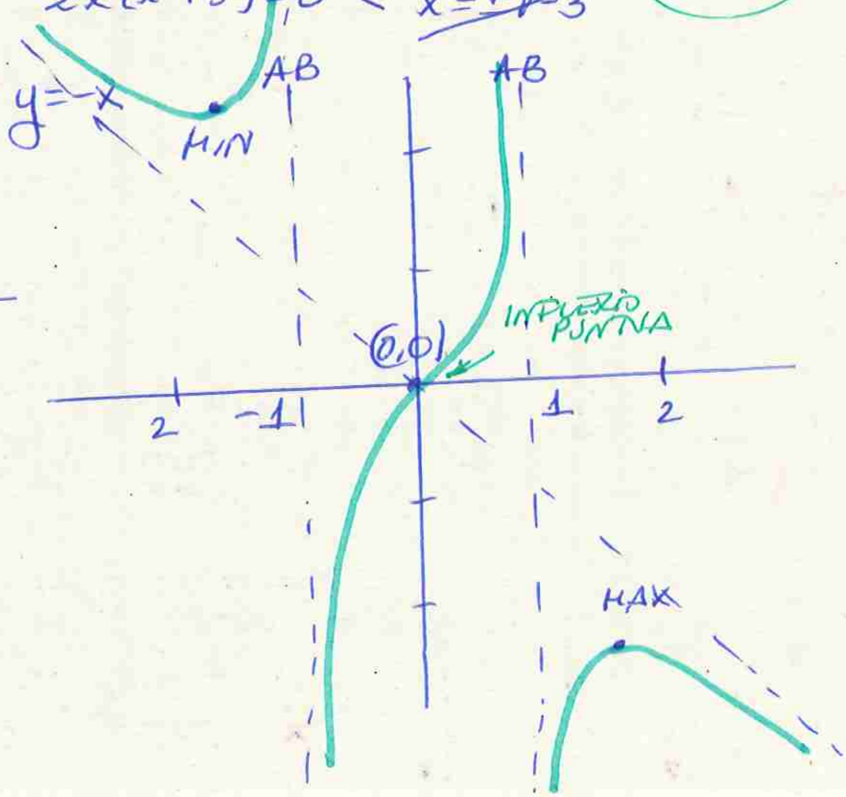
INFLEXIO PUNTUA $(0,0)$

$$\nexists f''(x) \longrightarrow x = \pm 1$$

| | $\frac{-}{-}$ | $-1$ | $\frac{-}{+}$ | $0$ | $\frac{+}{+}$ | $1$ | $\frac{+}{-}$ |
|---|---|---|---|---|---|---|---|
| $f''(x)$ | $f''>0$ | | $f''<0$ | | $f''>0$ | | $f''<0$ |
| $f(x)$ | $\cup$ | AB | $\cap$ | INF PUNT $(0,0)$ | $\cup$ | AB | $\cap$ |

$f''(-2) =$ $\qquad f''(2) = \frac{+}{-} = -$

$f''(-0{,}5) = \frac{-}{+} = -$



# ADIERAZPEN GRAFIKOA

## 11 b) $y = \dfrac{1}{x^2-1}$.

1./ DEFINIZIO EREMUA $\boxed{Dom f = \mathbb{R} - \{-1, 1\}}$

2./ EBAKETA PUNTUAK

OY ardatzo $x=0$ $y = \dfrac{1}{0-1} = -1 \longrightarrow \boxed{P(0,-1)}$.

OX ardatzo $y=0$ $0 = \dfrac{1}{x^2-1}$ Ez dau ebakitzen OX ardatzo

3/ SIMETRIA

$f(-x) = \dfrac{1}{(-x)^2-1} = \dfrac{1}{x^2-1} \longrightarrow f(x) = f(-x)$ $\boxed{\text{SIM. BIKOITIA OY ardatzarekiko}}$

4/ EZ DA PERIODIKOA

5/ ASINTOTAK

AB $\boxed{x=1 \quad x=-1}$

$x=1$ $\lim\limits_{x\to 1} \dfrac{1}{x^2-1} = \left(\dfrac{1}{0}\right)$

$\lim\limits_{\substack{x\to 1^-\\0{,}999}} \dfrac{1}{x^2-1} = \dfrac{1}{0{,}999^2-1} = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{\substack{x\to 1^+\\1{,}001}} \dfrac{1}{x^2-1} = \dfrac{1}{1{,}001^2-1} = \dfrac{1}{0^+} = +\infty$

$x=-1$ $\lim\limits_{x\to -1} \dfrac{1}{x^2-1} = \left(\dfrac{1}{0}\right)$

$\lim\limits_{\substack{x\to -1^-\\-1{,}001}} \dfrac{1}{(-1{,}001)^2-1} = \dfrac{1}{0^+} = +\infty$

$\lim\limits_{\substack{x\to -1^+\\-0{,}999}} \dfrac{1}{(-0{,}999)^2-1} = \dfrac{1}{0^-} = -\infty$

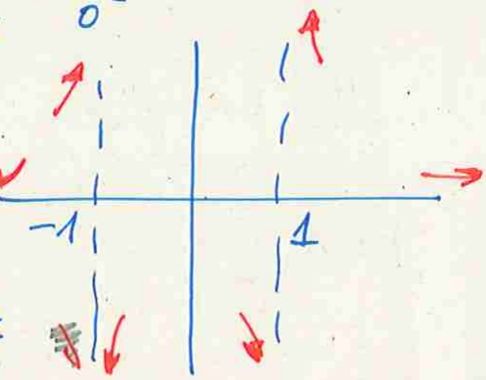
A.Horizont

$\lim\limits_{x\to\infty} \dfrac{1}{x^2-1} = 0. \longrightarrow \boxed{y=0}$ AH.

$f(x) = \dfrac{1}{x^2-1}$

$x \to +\infty$ $\dfrac{1}{1000^2-1} = +$ goitik $>0$.

$x \to -\infty$ $\dfrac{1}{(-1000)^2-1} = +$ goitik




6. HAZKUNDEA $y = (x^2-1)^{-1}$

$f'(x) = \dfrac{-2x}{(x^2-1)^2}$ $f'(x) = 0$ $\dfrac{-2x}{(x^2-1)^2} = 0$ $-2x = 0.$ $\boxed{x=0}$ Ptu sing.

$\nexists f'(x) \ni x = \pm 1$.

| | $-\infty$ ... $-1$ (AB) | $-1$ ... $0$ | $0$ ... $+1$ (AB) | $+1$ ... $+\infty$ |
|---|---|---|---|---|
| $f'(x)$ | $f'>0$ | $f'>0$ | $f'<0$ | $f'<0$ |
| $f(x)$ | ↗ | ↗ MAX $(0,-1)$ | ↘ | ↘ |

AB AB

GT $(-\infty,-1) \cup (-1,0)$

BT $(0,1) \cup (1,+\infty)$

Max $(0,-1)$

8/ AHURTASUNA / GANBILTASUNA

$$f'(x) = \frac{-2x}{(x^2-1)^2}$$

$$f''(x) = \frac{-2(x^2-1)^2 - (-2x)2(x^2-1)\cdot 2x}{(x^2-1)^4} =$$

$$= \frac{\cancel{(x^2-1)}\left[-2(x^2-1) + 8x^2\right]}{(x^2-1)^{\cancel{4}}} = \frac{-2x^2+2+8x^2}{(x^2-1)^3} =$$

$$f''(x) = \frac{6x^2+2}{(x^2-1)^3}$$

$$f''(x)=0 \longrightarrow \frac{6x^2+2}{(x^2-1)^3}=0 \;-\; 6x^2+2=0 \quad x=\sqrt{-\tfrac{1}{3}} \;\nexists x$$

$$\rightarrow \nexists f''(x) \quad x=\pm 1.$$

$f''(x) \neq 0$ Ez dugu aurkitzen P.Inf. baina kontuan hartzen dugu $f''(x)$ existitzen ez diren ~~puntuak~~ balioak.

| | | $x=-1$ | | $x=1$ | |
|---|---|---|---|---|---|
| $f''(x)$ | $f''>0$ | | $f''<0$ | | $f''>0$ |
| $f(x)$ | $\cup$ | | $\cap$ | | $\cup$ |