ADIERAZPEN GRAFIKOA

322) 13a) $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$.

1) DEFINIZIO EREMUA

$x^2 + x + 1 \neq 0 \qquad x = \dfrac{-1 \pm \sqrt{1^2 - 4\cdot 1}}{-1} = \dfrac{-1 \pm \sqrt{-3}}{-1} \quad \nexists x.$

$\boxed{\mathrm{Dom} f = \mathbb{R}.}$

2) EBAKETA PUNTUAK

OX ARDATZA $y = 0 \longrightarrow$ EZ DAGO EBAKETARIK

$0 = \dfrac{x^2 - x + 1}{x^2 + x + 1} \longrightarrow x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1-4}}{2} \quad \nexists x$

OY ARDATZA $x = 0$

$f(0) = \dfrac{0^2 - 0 + 1}{0^2 + 0 + 1} = 1 \longrightarrow \boxed{(0, 1)}$

3) SIMETRIA

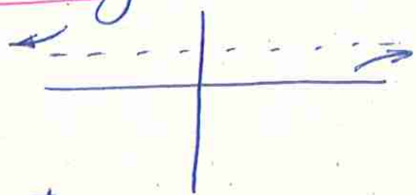
$f(-x) = \dfrac{(-x)^2 - (-x) + 1}{(-x)^2 - x + 1} = \dfrac{x^2 + x + 1}{x^2 - x + 1}$ EZ DA BAKOITIA eta EZ DA BIKOITIA $\rightarrow$ EZ DAUKA SIMETRIARIK

4) Ez da PERIODIKOA

5) ASINTOTAK

AB $\lim_{x \to ?} f(x) = \infty \qquad x^2 + x + 1 \neq 0 \rightarrow$ Ez dauka AB

AH $\lim_{x \to +\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} = \left(\dfrac{+\infty}{+\infty}\right) = 1 \longrightarrow \boxed{AH \Rightarrow y = 1}$



$\lim_{x \to -\infty} \dfrac{x^2 - x + 1}{x^2 + x + 1} \left(\dfrac{+\infty}{+\infty}\right) = 1$

$\lim_{x \to +\infty} f(x) = 1^-$ azpitik $\quad \lim_{x \to -\infty} f(x) = 1^+$ gainetik

Beste modu

$f(x) - A = \dfrac{x^2 - x + 1}{x^2 + x + 1} - 1 = \dfrac{\cancel{x^2} - x + 1 - (\cancel{x^2} + x + 1)}{x^2 + x + 1} = \dfrac{-2x}{x^2 + x + 1}.$

$\dfrac{-2x}{x^2 + x + 1} \begin{cases} x \to +\infty & \dfrac{-}{+} = \ominus \text{ azpitik} \\ x \to -\infty & \dfrac{+}{+} = \oplus \text{ gainetik} \end{cases}$

6/7) HAZKUNDEA ETA MUTUR ERLATIBOAK

$$f'(x)=\frac{(2x-1)(x^2+x+1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}=$$

$$=\frac{\cancel{2x^3}+\cancel{2x^2}+\cancel{2x}-\cancel{x^2}-\cancel{x}-1-\cancel{2x^3}-\cancel{x^2}+2x^2+\cancel{x}-\cancel{2x}-1}{(\quad)^2}=$$

$$\boxed{f'(x)=\frac{2x^2-2}{(x^2+x+1)^2}}$$

$f'(x)=0 \quad \frac{2x^2-2}{(\quad)^2}=0 \rightarrow 2x^2-2=0$

$x=\pm 1.$

$f'(x)$	$f'>0$	-1	$f'<0$	1	$f'>0$
$f(x)$	$\nearrow$	MAX $(-1,3)$	$\searrow$	MIN $(1,1/3)$	$\nearrow$

$f(-1)=3 \quad (-1,3)$

$f(1)=1/3 \quad (1,1/3)$

GT $(-\infty,-1)\cup(1,+\infty)$
BT $(-1,1)$

8) AHURTASUNA ETA GANBILTASUNA

$$f''(x)=\frac{4x(x^2+x+1)^{\cancel{2}}-(2x^2-2)2\cancel{(x^2+x+1)}\cdot(2x+1)}{(x^2+x+1)^{\cancel{4}\,3}}=$$

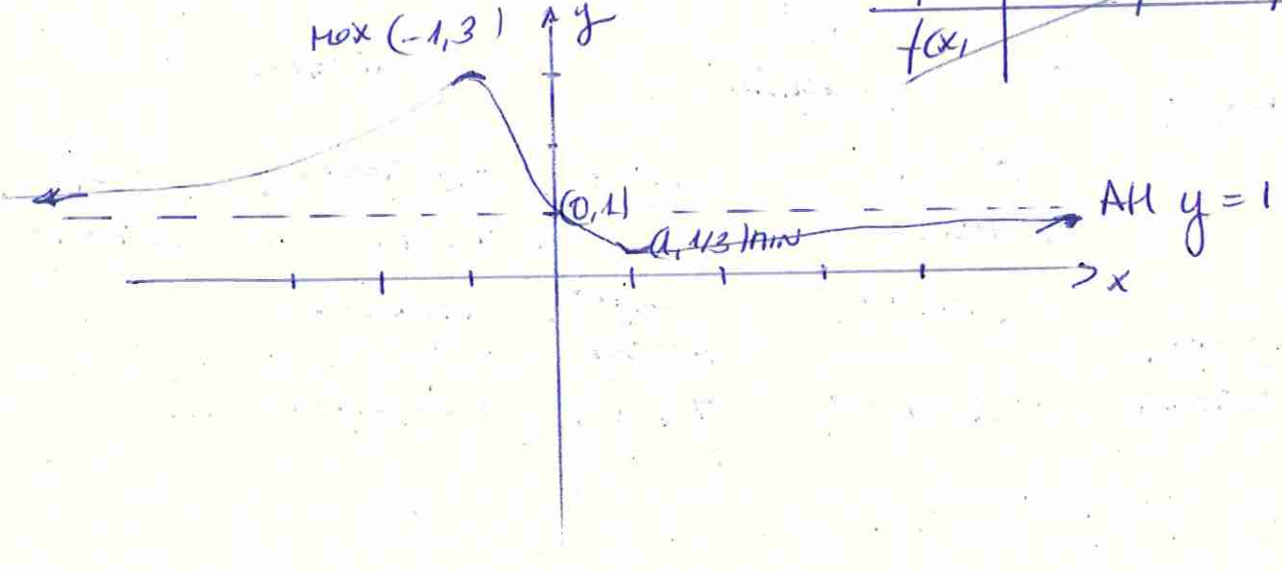
$$=\frac{4x(x^2+x+1)-2(2x^2-2)(2x+1)}{(x^2+x+1)^3}=$$

$$=\frac{4x^3+\cancel{4x^2}+4x-8x^3-\cancel{4x^2}+8x+4}{(\quad)^3}=\boxed{\frac{-4x^3+12x+4}{(x^2+x+1)^3}=f''(x)}$$

$f''(x)=0 \quad \frac{-4x^3+12x+4}{(x^2+x+1)^3}=0 \quad -4x^3+12x+4=0$ Ruffinipot ezin da

Ezin da kalkulatu IP.

$f''(x)$	-1	1
$f(x)$		



322) 136) $y=\dfrac{x^2-2x+2}{x-1}$

1) DEFINIZIO EREMUA $\boxed{Dom\, f = \mathbb{R}-\{1\}}$

2) EBAKETA PUNTUAK

OY ARDATZA $x=0$ $\quad y=\dfrac{0^2-2\cdot 0+2}{0-1}=-2$ $\quad \boxed{P(0,-2)}$

OX ARDATZA $y=0$ $\quad 0=\dfrac{x^2-2x+2}{x-1}$ $\quad x=\dfrac{2\pm\sqrt{2^2-4\cdot 2}}{2}\ \nexists\, x$

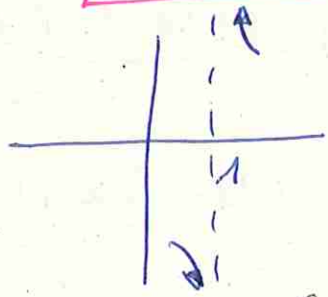
3) SIMETRIA

$f(-x)=\dfrac{(-x)^2-2(-x)+2}{(-x)-1}=\dfrac{x^2+2x+2}{-x-1}$ EZ DAUKA SIMETRIARIK

4) Ez da periodikoa

5) ASINTOTAK

$\boxed{AB}$ $\lim\limits_{x\to D} f(x)=\pm\infty.$ $\quad x-1\neq 0 \to \boxed{x=1 \text{ A. BERTIKALA}}$



$0{,}999 \rightarrow \ \leftarrow 1{,}001$; 1

$\lim\limits_{x\to 1^-} \dfrac{x^2-2x+2}{x-1}=\dfrac{1}{0^-}=-\infty$

$\lim\limits_{x\to 1^+} \dfrac{x^2-2x+2}{x-1}=\dfrac{1}{0^+}=+\infty$

$\boxed{AZ}$ dago $P(x)$ ren maila $>$ $Q(x)$ maila baino $(gP(x)=gQ(x)+1)$

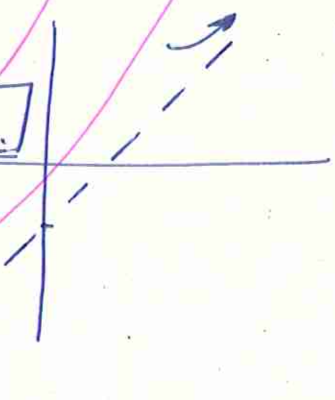
$$\begin{array}{l|l} x^2-2x+2 & x-1 \\ \hline -x^2+x & x-1 \\ \quad -x+2 & \\ \quad +x-1 & \\ \hline \quad\quad 1 & \end{array}$$

$f(x)=\underbrace{x-1}_{AZ}+\underbrace{\dfrac{1}{x-1}}_{\text{distantzia}}$ $\quad$ AZ $\boxed{y=x-1}$

$f(x)-(x-1)=\dfrac{1}{x-1}$

$x\to+\infty \quad \dfrac{1}{x-1}=(+)$ funtzioa asintotaren GAINETIK

$x\to-\infty \quad \dfrac{1}{x-1}=(-)$ funtzioa asintotaren AZPITIK



Beste modu batera $\boxed{m=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^2-2x+2}{x^2-x}=1}$

$n=\lim\limits_{x\to\infty}(f(x)-mx)=\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+2}{x-1}-x\right)=$

$=\lim\limits_{x\to\infty}\dfrac{x^2-2x+2-x^2+x}{x-1}=\lim\limits_{x\to\infty}\dfrac{-x+2}{x-1}=\boxed{-1}$

6,7) HAZKUNDEA ETA MUTUR ERLATIBOAK

$$f(x)=\frac{x^2-2x+2}{x-1}$$

$$f'(x)=\frac{(2x-2)(x-1)-(x^2-2x+2)}{(x-1)^2}=\frac{2x^2-\cancel{2x}-2x+\cancel{2}-x^2+\cancel{2x}-\cancel{2}}{(x-1)^2}$$

$$f'(x)=\frac{x^2-2x}{(x-1)^2} \qquad f'(x)=0 \longrightarrow 0=\frac{x^2-2x}{(x-1)^2} \qquad \boxed{\begin{matrix}x_1=0\\ x_2=2\end{matrix}}$$

				AB			
$f'(x)$	$f'>0$	0	$f'<0$	1	$f'<0$	2	$f'<0$
$f(x)$	↗	MAX $(0,-2)$	↘		↘	MIN $(2,2)$	↗

GT $(-\infty,0)\cup(2,+\infty)$
BT $(0,1)\cup(1,2)$
Max $(0,-2)$
Min $(2,2)$

8) AHURT/GANBILT.

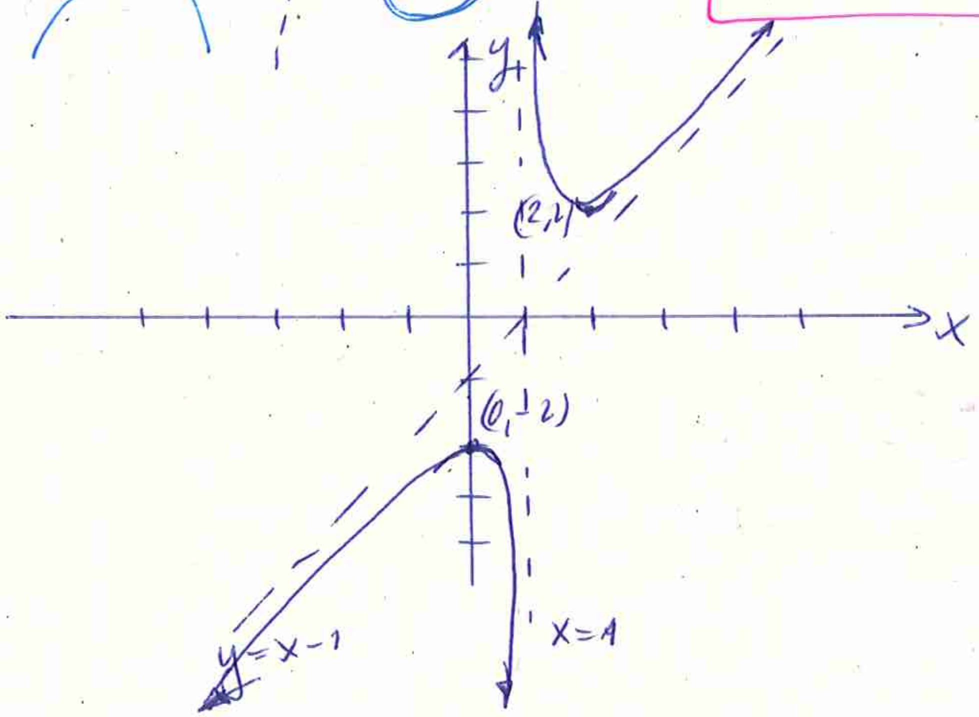
$$f''(x)=\frac{(2x-2)(x-1)^{\cancel{2}}-2\cancel{(x-1)}(x^2-2x)}{(x-1)^{\cancel{4}\,3}}$$

$$f''(x)=\frac{(2x-2)(x-1)-2(x^2-2x)}{(x-1)^3}=\frac{\cancel{2x^2}-\cancel{2x}-\cancel{2x}+2-\cancel{2x^2}+\cancel{4x}}{(\quad)^3}$$

$$f''(x)=\frac{2}{(x-1)^3} \qquad \nexists x \,/\, f''(x)=0.$$ Ez dago inflexio-puntuik.

		AB	
$f''(x)$	$f''<0$	1	$f''>0$
$f(x)$	∩		∪

AHURRA $(1,2)\cup(2,+\infty)$
GANBILA $(-\infty,0)\cup(0,1)$



14a) $y=\sqrt[3]{4-x^2}$

1) Definizio eremua $\quad Dom f=\mathbb{R}.$

2) EBAKETA PUNTUAK

OY ARDATZA $\quad x=0 \longrightarrow y=\sqrt[3]{4} \quad \boxed{P(0;1{,}58)}$

OX ARDATZA $\quad y=0 \longrightarrow 0=\sqrt[3]{4-x^2}$

$0^3=4-x^2 \longrightarrow x^2=4 \begin{cases} x_1=2 \\ x_2=-2 \end{cases} \quad \boxed{\begin{matrix}(2,0)\\(-2,0)\end{matrix}}$

3) Ez da PERIODIKOA

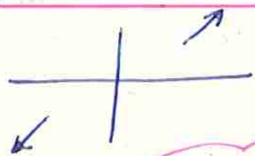
4) SIMETRIA:

$f(-x)=\sqrt[3]{4-(-x)^2}=\sqrt[3]{4-x^2} \quad f(x)=f(-x)$ $\boxed{\text{SIMETRIA BIKOITIA}}$ y ardatzarekiko

5) ASINTOTAK

AB $\lim\limits_{x\to ?} f(x)=\infty \longrightarrow$ Ez dauka AB.

Aztertuz: $\left.\begin{matrix}\lim\limits_{x\to+\infty} f(x)=-\infty \\ \lim\limits_{x\to-\infty} f(x)=+\infty\end{matrix}\right\}$ Adar parabolikoak.



6) HAZKUNDEA eta MUTUR ERLATIBOAK $\quad$ Kalkulatu $f'=0$ eta $\nexists f'=0$

$f'(x)=\dfrac{-2x}{3\sqrt[3]{(4-x^2)^2}} \qquad f'(x)=0 \longrightarrow -2x=0 \longrightarrow \boxed{x=0}$

$\nexists f'(x) \longrightarrow x=\pm2$

7) AHURT. eta GANBILTASUNA $\quad$ Kalkulatu $f''=0$ eta $\nexists f''$

$f'(x)=-\dfrac{2}{3}\left(x\cdot\sqrt[3]{(4-x^2)^2}\right)=\dfrac{-2}{3}x\cdot(4-x^2)^{-2/3}$

$f''(x)=-\dfrac{2}{3}\left[(4-x^2)^{-2/3}-\dfrac{2}{3}x\,(4-x^2)^{-5/3}(-2x)\right]=$

$=-\dfrac{2}{3\sqrt[3]{(4-x^2)^2}}-\dfrac{8x^2}{9(4-x^2)\sqrt[3]{(4-x^2)^2}}=\dfrac{-6(4-x^2)-8x^2}{3(4-x^2)\sqrt[3]{(4-x^2)^2}}$

$=\dfrac{-24-2x^2}{\underbrace{3(4-x^2)\sqrt[3]{(4-x^2)^2}}_{=0\,\rightarrow\, x=\pm2}}$

$f''(x)=0.$

$-24-2x^2=0$

$x=\sqrt{-12} \quad \nexists x.$

Ez dago infl. puntuak.

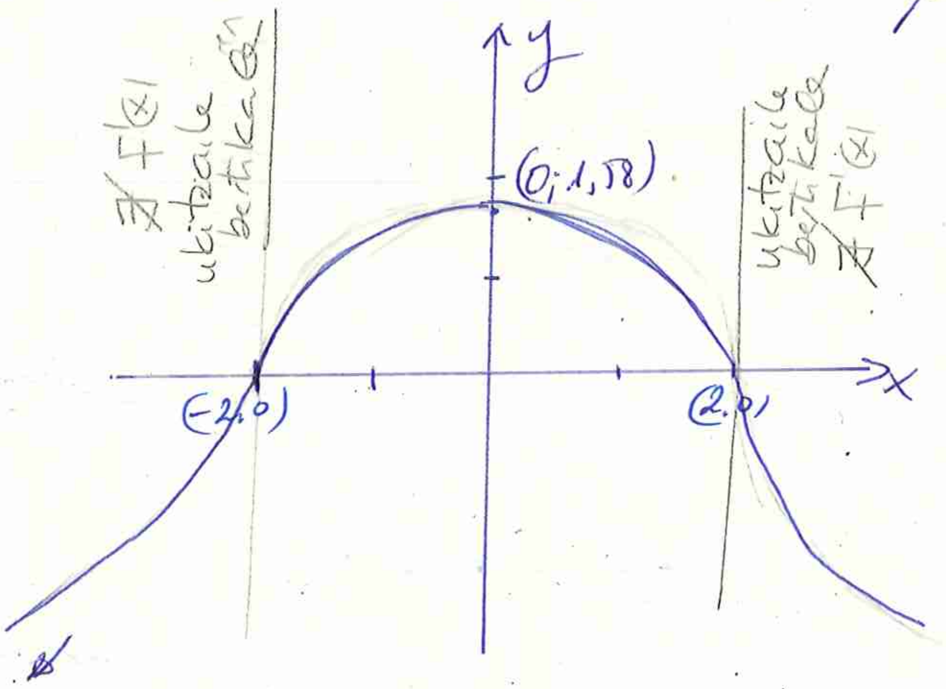
HAZKUNDEA

$f'(x)=0 \longrightarrow x=0. \quad f(0)=\sqrt[3]{4}=1,58 \quad (0;1,58)$

$f''(0)=\dfrac{-24}{3\cdot 4\sqrt[3]{4}}<0$ ganbila denez MAXIMOA da.

$f'(x)$	$f'>0$	$\cancel{\exists}$ -2	$f'>0$	0	$f'<0$	$\cancel{\exists}$ 2	$f'<0$
$f(x)$	↗		↗	$(0;1,58)$ MAX	↘		↘

$f''(x)$	$f''>0$	-2	$f''<0$	2	$f''>0$
$f(x)$	∪	INF PUNT.	∩	INF PUNT.	∪



15) $y = \dfrac{x}{e^x}$

Dom $f = \mathbb{R}$.

2/ Ebaketa puntuak: OY ardatza $x=0$ $y=0$ $P(0,0)$; OX ardatza $y=0 \to x=0$

3/ Ez da periodikoa

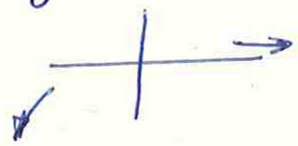
4/ Ez da simetrikoa

5/ Asintotak.

AB ez dago

AH, AZ? $\lim_{x\to+\infty} f(x) = \dfrac{+\infty}{+\infty} = 0. \to \boxed{y=0.}$ AH

$\lim_{x\to-\infty} f(x) = \dfrac{-\infty}{0} = -\infty$



6/ Hazkundea eta muturrak
7

$y' = \dfrac{e^x - e^x \cdot x}{e^{2x}} = \dfrac{e^x(1-x)}{e^{2x}} = \dfrac{1-x}{e^x}.$

$f'(x) = 0 \longrightarrow 1-x=0 \quad \boxed{x=1}$

$f'(x)$	$f'>0$	1	$f'<0$
$f(x)$	$\nearrow$	$\boxed{\left(1, \frac{1}{e}\right)}$ MAX.	$\searrow$

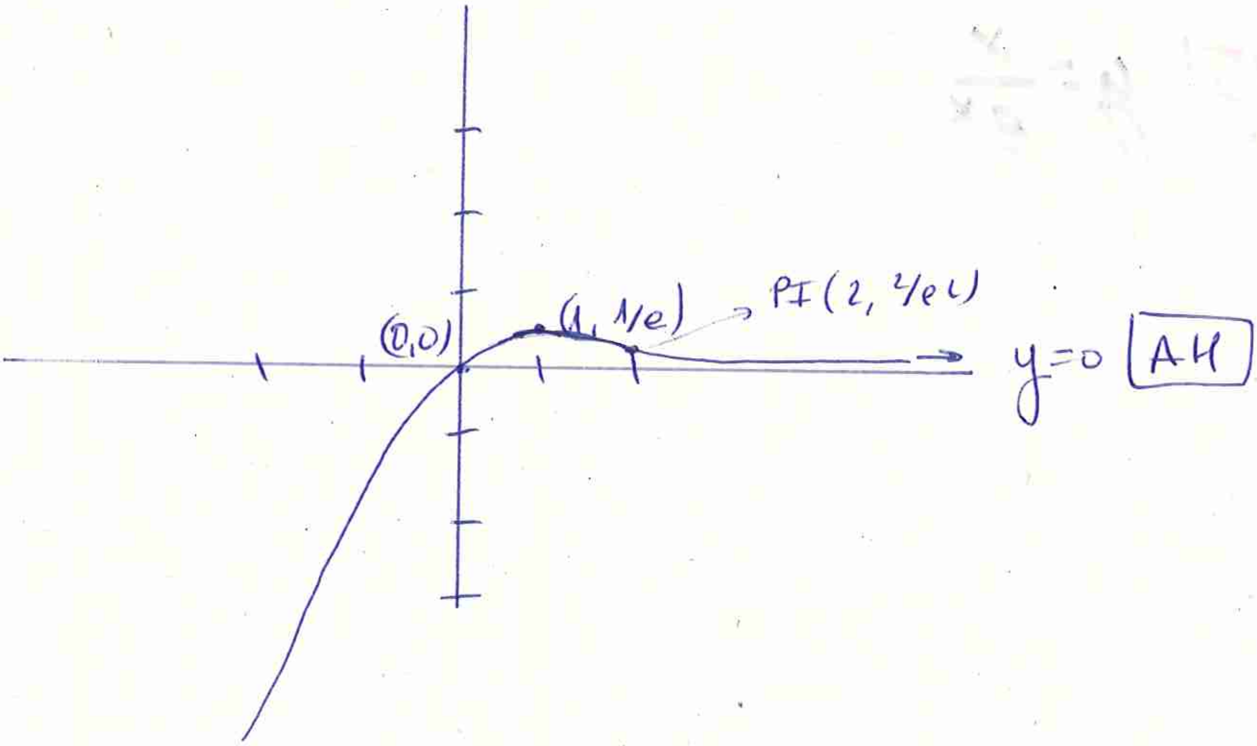
G.T $(-\infty, 1)$
BT $(1, +\infty)$
MAX $(1, 1/e)$
AHURRA $(2, +\infty)$
GANBILA $(-\infty, 2)$
INF PUNT $(2, 2/e^2)$

8/ Ahurra/ganbila.

$f''(x)$	$f''<0$	2	$f''>0$
$f(x)$	$\frown$	$\boxed{(2, 2/e^2)}$ INFLEX	$\smile$

$f''(x) = \dfrac{-1 \cdot e^x - (1-x)\cdot e^x}{e^{2x}} = \dfrac{e^x(-1-1+x)}{e^{2x}} = \dfrac{x-2}{e^x}$

$f''(x) = 0 \quad x-2=0 \quad \underline{x=2.}$ $\boxed{IP.\ (2, 2/e^2)}$



$$\lim_{x\to+\infty} \frac{x}{e^x} = 0. \text{ AH.}$$

$$\lim_{x\to-\infty} \frac{x}{e^x} = \frac{-\infty}{0} = -\infty$$

$$\lim_{x\to-\infty} \frac{\cancel{x}/e^x}{\cancel{x}} = \lim_{x\to-\infty} \frac{1}{e^x} = \lim_{x\to+\infty} \frac{1}{e^{-x}} =$$

$$= \lim_{x\to+\infty} e^x = +$$