LIMITEAK.

225) 1]

a) $\lim\limits_{x \to -1} \dfrac{x^3+2x^2+x}{x^3+x^2-x-1} = \left(\dfrac{0}{0}\right)$

① Faktorizatzen:

$\lim\limits_{x \to -1} \dfrac{x\cdot\cancel{(x+1)^2}}{(x-1)\cancel{(x+1)^2}} = \lim\limits_{x \to -1} \dfrac{x}{x-1} = \boxed{\dfrac{1}{2}}$

② L'Hopital

$\lim\limits_{x \to -1} \dfrac{3x^2+4x+1}{3x^2+2x-1} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x \to -1} \dfrac{6x+4}{6x+2} =$

$= \dfrac{-2}{-4} = \boxed{\dfrac{1}{2}}$

b) $\lim\limits_{x \to 0} \dfrac{e^{-x}+x-1}{x^2} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x \to 0} \dfrac{-e^{-x}+1}{2x} = \left(\dfrac{0}{0}\right)$

$\overset{H}{=} \lim\limits_{x \to 0} \dfrac{e^{-x}}{2} = \boxed{\dfrac{1}{2}}$

c) $\lim\limits_{x \to 0} \dfrac{\sin x\,(1+\cos x)}{x\cdot\cos x} = \left(\dfrac{0}{0}\right) =$

$\overset{H}{=} \lim\limits_{x \to 0} \dfrac{\cos x\,(1+\cos x) + \cancel{\sin^2 x}^{\,0}}{\cos x + \cancel{x\,\sin x}_{\,0}} = 2.$

d) $\lim\limits_{x \to 0} \dfrac{e^{x}-e^{-x}}{\sin x} = \left(\dfrac{0}{0}\right) = \lim\limits_{x \to 0} \dfrac{e^{x}+e^{-x}}{\cos x} = 2.$

e) $\lim_{x\to 0} (\cos x + \sin x)^{1/x} = 1^{\infty}$

① $e^{\lim_{x\to 0} (\cos x + \sin x - 1)\frac{1}{x}} = e$

$$\lim_{x\to 0} (\cos x + \sin x - 1)\frac{1}{x} = (0\cdot\infty)$$

$$= \lim_{x\to 0} \frac{\cos x + \sin x - 1}{x} = \left(\frac{0}{0}\right)$$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{-\sin x + \cos x}{1} = 1$$

$$\Rightarrow e^1 = \boxed{e}$$

② Beste modu batera → LOGARITMOEKIN.

$$\cancel{\lim_{x\to 0} \lim_{x\to 0} \ln f(x) = \lim_{x\to 0} \frac{1}{x}}$$

$$\lim_{x\to 0} \ln f(x) = \lim_{x\to 0} \ln (\cos x + \sin x)^{1/x} =$$

$$= \lim_{x\to 0} \frac{1}{x} \ln(\cos x + \sin x) = \left(\frac{0}{0}\right) \stackrel{H}{=}$$

$$= \lim_{x\to 0} \frac{\frac{-\sin x + \cos x}{\cos x + \sin x}}{1} = 1$$

$$\Rightarrow \lim_{x\to 0} f(x) = e^1 = \boxed{e}$$

f) $\lim_{x\to+\infty} (1-2^{1/x})x = 0\cdot\infty$

Bihurtu zatiketa:

$$\lim_{x\to+\infty} \frac{1-2^{1/x}}{1/x} = \left(\frac{0}{0}\right) \overset{H}{=} \lim_{x\to+\infty} \frac{-2^{1/x}\cancel{(-1/x^2)}\ln 2}{\cancel{(-1/x^2)}} =$$

$$= \lim_{x\to+\infty} -2^{1/x}\cdot\ln 2 = -\ln 2 = \ln 2^{-1} = \boxed{\ln\frac{1}{2}}$$

g) $\lim_{x\to 2} (3-x)^{\frac{2}{x^2-4}} = 1^{\infty}$

Formulagaz: $e^{\lim_{x\to 2} (3-x-1)\cdot\frac{2}{x^2-4}} =$

$$\lim_{x\to 2} \frac{\cancel{(2-x)}\cdot 2}{(x+2)\cancel{(x-2)}} = \lim_{x\to 2} \frac{-2}{x+2} = -\frac{1}{2}$$

$$\Rightarrow e^{-1/2}$$

Logaritmoekin $\lim_{x\to 2} \ln f(x) = \cancel{\lim_{x\to 2} \ln (3-x)^{\frac{2}{x^2-4}}} =$

$$\lim_{x\to 2} \ln (3-x)^{\frac{2}{x^2-4}} = \lim_{x\to 2} \frac{2}{x^2-4}\ln(3-x) = \left(\frac{0}{0}\right)$$

$$\overset{H}{=} \lim_{x\to 2} \frac{\frac{-1}{3-x}}{2x} = -\frac{1}{2} \Rightarrow \boxed{e^{-1/2}}$$

h.) $\lim_{x\to 5} \frac{\sqrt{x^2-9}-4}{x-5} = \left(\frac{0}{0}\right) \overset{H}{=} \lim_{x\to 5} \frac{\frac{2x}{2\sqrt{x^2-9}}}{1} = \frac{10}{2\sqrt{25-9}} = \frac{5}{4}$