242. orrialdea **17. ariketa** L'HOPITAL

a) $\lim\limits_{x\to -1} \dfrac{x^3+1}{x^2-3x-4} = \dfrac{-1+1}{1+3-4} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to -1} \dfrac{3x^2}{2x-3} = \boxed{\dfrac{3}{-5}}$

Beste modu bat → FAKTORIZATZEN ETA SINPLIFIKATZEN.

b) $\lim\limits_{x\to 0} \dfrac{\ln(e^x+x^3)}{x} = \dfrac{\ln(\overset{1}{e^0}+0^3)}{0} = \left(\dfrac{0}{0}\right) \overset{H}{=}$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{e^x+3x^2}{e^x+x^3}}{1} = \lim\limits_{x\to 0} \dfrac{e^x+3x^2}{e^x+x^3} = \dfrac{e^0+\cancel{3\cdot 0^2}}{e^0+0^3} = \boxed{1}$

c) $\lim\limits_{x\to 0} \dfrac{\sin x}{1-\cos x} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{-(-\sin x)} = \left(\dfrac{1}{0}\right)$

$-0{,}01 \quad +0{,}01$; 0

$\begin{cases} \lim\limits_{x\to 0^-} \dfrac{\cos x}{\sin x} = \dfrac{1}{0^-} = -\infty \\ \lim\limits_{x\to 0^+} \dfrac{\cos x}{\sin x} = \dfrac{1}{0^+} = +\infty \end{cases}$

Ez da existitzen limitea.

d) $\lim\limits_{x\to 0} \dfrac{a^x-b^x}{x} = \dfrac{a^0-b^0}{0} = \dfrac{1-1}{0} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{a^x\ln a - b^x \ln b}{1}$

$= a^0\ln a - b^0 \ln b = \ln a - \ln b = \underline{\underline{\ln(a/b)}}$

e) $\lim\limits_{x\to 0} \dfrac{\operatorname{arctg} x - x}{x-\sin x} = \dfrac{\operatorname{arctg} 0 - 0}{0-\sin 0} = \left(\dfrac{0}{0}\right) =$

* Deribatu $(1+x^2)^{-1}$

$\overset{H}{=} \lim\limits_{x\to 0} \dfrac{\dfrac{1}{1+x^2}-1}{1-\cos x} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\dfrac{-2x}{(1+x^2)^2}}{\sin x} = \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{-2x}{(1+x^2)^2\cdot \sin x} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-2}{\underbrace{2(1+x^2)\cdot 2x\cdot \sin x}_{0} + \underbrace{\cos x\cdot (1+x^2)^2}_{1}}$

$= \dfrac{-2}{1} = \boxed{-2}$

f) $\lim\limits_{x\to 0} \dfrac{e^x - e^{\sin x}}{1-\cos x} = \dfrac{e^0 - e^0}{1-\cos 0} = \left(\dfrac{0}{0}\right) \overset{H}{=}$

$\overset{H}{=} \lim\limits_{x\to 0} \dfrac{e^x - e^{\sin x}\cdot \cos x}{-(-\sin x)} = \dfrac{e^0 - e^0 \cdot 1}{\sin 0} = \left(\dfrac{0}{0}\right) \overset{H}{=}$

$= \lim\limits_{x\to 0} \dfrac{e^x - (e^{\sin x}\cdot \cos x \cdot \cos x + e^{\sin x}\cdot(-\sin x))}{\cos x} =$

$= \lim\limits_{x\to 0} \dfrac{e^x - e^{\sin x}\cos^2 x + e^{\sin x}\sin x}{\cos x} = \dfrac{e^0 - e^0 \cdot 1 + e^0 \cdot 0}{1}$

$= 0/1 = 0$

g) $\lim\limits_{x\to 0} \dfrac{\ln(\cos 3x)}{x^2} = \left(\dfrac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{\frac{-\sin(3x)\cdot 3}{\cos(3x)}}{2x} =$

$= \lim\limits_{x\to 0} \dfrac{-3\operatorname{tg}(3x)}{2x} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-3\cdot 3\cdot(1+\operatorname{tg}^2 x)}{2} =$

$= \boxed{-9/2}$

h) $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{\sqrt[4]{x^3}}$ $x^{3/4}$ $= \dfrac{\ln 1}{0} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{1+x}}{\frac{3}{4}x^{3/4-1}} =$

$x^{-1/4} = \dfrac{1}{\sqrt[4]{x}}$

$= \lim\limits_{x\to 0} \left(\dfrac{1}{1+x} : \dfrac{3}{4\sqrt[4]{x}}\right) = \lim\limits_{x\to 0} \dfrac{4\sqrt[4]{x}}{3(1+x)} = \dfrac{0}{3} = 0$

i) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(2x)}{3x^2} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-\cancel{2}\cos(2x)(-\sin 2x)\cdot 2}{3\,\cancel{2}x}$

$= \lim\limits_{x\to 0} \dfrac{2\sin(2x)\cdot\cos(2x)}{3x} = \lim\limits_{x\to 0} \dfrac{\sin 4x}{3x} = \left(\dfrac{0}{0}\right) \overset{H}{=}$

FORMULA TRIGONOMETRIKOA
$\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha$.

$\overset{H}{=} \lim\limits_{x\to 0} \dfrac{4\cdot\cos 4x}{3} = \boxed{\dfrac{4}{3}}$