242) 12

a) $\lim\limits_{x\to 1} \dfrac{x^2-7x+6}{1-x} = \left(\dfrac{0}{0}\right)$ IND.

$\left(\dfrac{0}{0}\right)$ Faktorizáljuk a tört:

$x^2-7x+6=0$

$x=\dfrac{7\pm\sqrt{7^2-4\cdot 6}}{2}=\dfrac{7\pm 5}{2}$

$x_1=6$

$x_2=1$

$\lim\limits_{x\to 1} \dfrac{(x-6)(x-1)}{1-x} = \lim\limits_{x\to 1} \dfrac{(x-6)(x-1)}{-(x-1)} =$

$= \lim\limits_{x\to 1} (6-x) = 5.$

b) $\lim\limits_{x\to 1} \dfrac{x^3-4x^2+5x-2}{(x^3-1)(x-2)} = \left(\dfrac{0}{0}\right)$ IND.

$\lim\limits_{x\to 1} \dfrac{(x-1)(x-2)(x-1)}{(x-1)(x^2+x+1)(x-2)} =$

$= \lim\limits_{x\to 1} \dfrac{x-1}{x^2+x+1} = \dfrac{0}{3} = 0$

x^3-4x^2+5x-2

	1	−4	+5	−2
1		1	−3	2
	1	−3	2	0

$x^2-3x+2=0$

$x=\dfrac{3\pm\sqrt{3^2-4\cdot 2}}{2}=\dfrac{3\pm 1}{2}$

$x_1=2$ $x_2=1$

x^3-1

	1	0	0	−1
1		1	1	1
	1	1	1	0

$(x-1)(x^2+x+1)$

c) $\lim\limits_{x\to 2/3} \dfrac{3x^2+5x+2}{9x^2-4} = \dfrac{2/3}{0}$

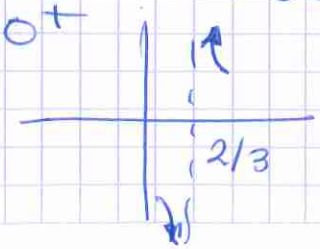
$x=\dfrac{-5\pm\sqrt{5^2-4\cdot 3\cdot 2}}{2\cdot 3}=\dfrac{-5\pm 1}{6}$

$x_1=-1$ $x_2=-2/3$

$\lim\limits_{x\to 2/3} \dfrac{3(x+1)(x+2/3)}{3(x+2/3)(x-2/3)} = \left(\dfrac{15}{0}\right)$ Alsó limiteak.

$\lim\limits_{x\to 2/3^-} \dfrac{x+1}{3(x-2/3)} = \dfrac{5/3}{0^-} = -\infty$

$\lim\limits_{x\to 2/3^+} \dfrac{x+1}{3(x-2/3)} = \dfrac{5/3}{0^+} = +\infty$



$\lim_{x \to 2/3^+} \frac{x+1}{3(x-2/3)} = \frac{5/3}{0^+} = +\infty$

	$2/3$	
	↓	
$+\infty$		$-\infty$

d) $\lim_{x \to 2} \frac{x^2+3x-10}{x^3-x^2-8x+12}$

$x^2+3x-10$ $\quad x = \frac{-3 \pm \sqrt{3^2-4 \cdot 1 \cdot (-10)}}{2} = \frac{-3 \pm 7}{2} = \begin{cases} -5 \\ 2 \end{cases}$

$x^3-x^2-8x+12$

	1	−1	−8	12
(2)		2	2	−12
	1	1	−6	0

$x^2+x-6=0$

$x = \frac{-1 \pm \sqrt{1^2-4 \cdot 1 \cdot (-6)}}{2}$

$x_1 = (2) \quad x_2 = (-3)$

$\lim_{x \to 2} \frac{(x-2)(x+5)}{(x-2)(x-2)(x+3)} = \left(\frac{7}{0}\right)$ Albo limita

		4
	↓	
$-\infty$		$+\infty$

→		←
1.999	2	2.001

$\lim_{x \to 2^-} \frac{x+5}{(x-2)(x+3)} = \frac{7}{0^-} = -\infty$

$\lim_{x \to 2^+} \frac{x+5}{(x-2)(x+3)} = \frac{7}{0^+} = +\infty$