243. orrialdea. $\boxed{\text{24. ariketa}}$

a) $\lim_{x\to+\infty} \left(\dfrac{2x^3+x^2-3}{5x^3-2x^2}\right)^{1-x} = \left(\dfrac{+\infty}{+\infty}\right)^{-\infty} = \left(\dfrac{2}{5}\right)^{-\infty} = \left(\dfrac{5}{2}\right)^{+\infty} = +\infty$

Maila berdineko polinomioak $\rightarrow \lim_{x\to+\infty} f(x) = \dfrac{a}{b}$.

b) $\lim_{x\to+\infty} \left(\dfrac{2x-5}{2x+3}\right)^{\frac{x+1}{2}} = (1^{\infty})$

Formula erabiliz $\lim_{x\to+\infty} f(x)^{g(x)} = e^{\underbrace{\lim_{x\to+\infty}(f(x)-1)\cdot g(x)}_{*}}$

$\lim_{x\to+\infty} (f(x)-1)\cdot g(x) = \lim_{x\to+\infty} \left(\dfrac{2x-5}{2x+3}-1\right)\cdot\dfrac{x+1}{2} =$

$= \lim_{x\to+\infty} \dfrac{\cancel{2x}-5-(\cancel{2x}+3)}{2x+3}\cdot\dfrac{x+1}{2} = \lim_{x\to+\infty} \dfrac{-8(x+1)}{2(2x+3)} =$

$= \lim_{x\to+\infty} \dfrac{-8x-8}{4x+6} = \left(\dfrac{-\infty}{+\infty}\right) = \dfrac{-8}{4} = \boxed{-2}$.

Beraz:

$\lim_{x\to+\infty} f(x)^{g(x)} = \boxed{e^{-2}}$

c) $\lim_{x\to 0} \dfrac{x\cdot\sin x}{1-\cos x} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim_{x\to 0} \dfrac{\sin x + x\cos x}{\sin x} = \left(\dfrac{0}{0}\right) \overset{H}{=}$

$\overset{H}{=} \lim_{x\to 0} \dfrac{\cos x+\cos x - x\cdot\sin x}{\cos x} = \dfrac{\overset{1}{\cos 0}+\overset{1}{\cos 0}-0\cdot\overset{0}{\sin 0}}{\underbrace{\cos 0}_{1}} = \underline{\underline{2}}$

d) $\lim_{x\to 2} \dfrac{\sqrt{13-x^2}-3}{x-2} = \left(\dfrac{0}{0}\right)$ Bi modutara egin daiteke

1) L'HOPITAL.

$\overset{H}{=} \lim_{x\to 2} \dfrac{\frac{-\cancel{2}x}{\cancel{2}\sqrt{13-x^2}}}{1} = \lim_{x\to 2} -\dfrac{x}{\sqrt{13-x^2}} = \boxed{-\dfrac{2}{3}}$

2) KONJUKANAREKİN BİDERKATVZ $(a-b)(a+b) = a^2 - b^2$

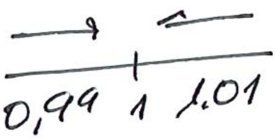
$$\lim_{x\to 2} \frac{\sqrt{13-x^2}-3}{x-2} \cdot \frac{\sqrt{13-x^2}+3}{\sqrt{13-x^2}+3} = \lim_{x\to 2} \frac{(\sqrt{13-x^2})^2 - 3^2}{(x-2)(\sqrt{13-x^2}+3)} =$$

$$= \lim_{x\to 2} \frac{13-x^2-9}{(x-2)(\sqrt{13-x^2}+3)} = \lim_{x\to 2} \frac{4-x^2}{(x-2)(\sqrt{13-x^2}+3)} = \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 2} \frac{(2+x)\overset{-1}{\cancel{(2-x)}}}{\cancel{(x-2)}(\sqrt{13-x^2}+3)} \qquad \frac{2-x}{x-2} = \frac{-x+2}{x-2} = \frac{-\cancel{(x-2)}}{\cancel{x-2}} = -1$$

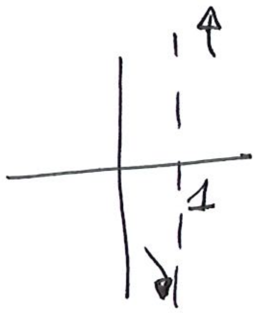
$$= \lim_{x\to 2} \frac{-(2+x)}{\sqrt{13-x^2}+3} = \frac{-4}{6} = \boxed{-2/3}$$

e) $\lim_{x\to 1} \frac{\ln x}{(x-1)^2} = \left(\frac{0}{0}\right) \overset{H}{=} \lim_{x\to 1} \frac{1/x}{2(x-1)} = \left(\frac{1}{0}\right)$



$$\lim_{x\to 1^-} \frac{1/x}{2(x-1)} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 1^+} \frac{1/x}{2(x-1)} = \frac{1}{0^+} = +\infty$$



f) $\lim_{x\to -\infty} x^2 \cdot e^{-x} = \lim_{x\to +\infty} (-x)^2 e^{-(-x)} = \lim_{x\to +\infty} x^2 \cdot e^{x} = \infty \cdot \infty$

$$= \boxed{+\infty}$$

g) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \frac{1}{0} - \frac{1}{0} = (\pm\infty - (\pm\infty))$ IND

$$= \lim_{x\to 0} \frac{\sin x - x}{x \cdot \sin x} = \frac{\sin 0 - 0}{0 \cdot \sin 0} = \left(\frac{0}{0}\right) = H$$

$$\overset{H}{=} \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x \cdot \cos x} = \frac{\overset{1}{\cancel{\cos 0}} - 1}{\underset{0}{\cancel{\sin 0}} + 0 \cdot \cancel{\cos 0}} = \left(\frac{0}{0}\right) \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{-\sin x}{\cos x + \cos x - x \sin x} = \lim_{x\to 0} \frac{-\sin x}{2 \cdot \cos x - x \sin x} =$$

$$= \frac{-\sin 0}{2 \cdot \underset{1}{\cancel{\cos 0}} - 0 \cdot \sin 0} = \frac{0}{2} = \underline{\underline{0}}$$

$$h)\ \lim_{x\to 0} \frac{e^x - x\cos x - 1}{\sin x - x + 1 - \cos x} = \frac{e^0 - \cancel{0\cdot\cos 0} - 1}{\cancel{\sin 0} - 0 + 1 - \underbrace{\cos 0}_{1}} = \left(\frac{0}{0}\right)$$

$$\overset{H}{=} \lim_{x\to 0} \frac{e^x - (\cos x + x(-\sin x))}{\cos x - 1 - (-\sin x)} = \lim_{x\to 0} \frac{e^x - \cos x + x\sin x}{\cos x - 1 + \sin x}$$

$$= \frac{e^0 - \cos 0 + 0.\sin 0}{\cos 0 - 1 + \sin 0} = \frac{1-1+0}{1-1+0} = \left(\frac{0}{0}\right) \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to 0} \frac{e^x + \sin x + \sin x + x\cos x}{-\sin x + \cos x} = \frac{e^0 + 2\overset{0}{\cancel{\sin 0}} + \overset{0}{\cancel{0.\cos 0}}}{-\underbrace{\cancel{\sin 0}}_{0} + \underbrace{\cos 0}_{1}}$$

$$= 1/1 = \boxed{1}$$

j) $\lim\limits_{x\to 0} \dfrac{x-\sin x}{x\cdot \sin x} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin x + x\cdot\cos x} =$

$= \dfrac{1-1}{0} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{\cos x + \cos x + x(-\sin x)} =$

$= \lim\limits_{x\to 0} \dfrac{\sin x}{2\cos x - x\sin x} = \dfrac{0}{1} = 0$

k) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{e^x - 1} = \left(\dfrac{0}{0}\right) \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-(-\sin x)}{e^x} = \dfrac{0}{1} = 0$

l) $\lim\limits_{x\to \pi/2} \dfrac{\operatorname{tg} x - 8}{\sec x + 10} = \left(\dfrac{+\infty}{+\infty}\right) \overset{H}{=} \lim\limits_{x\to \frac{\pi}{2}} \dfrac{\frac{1}{\cos^2 x}}{-1\cdot(\cos x)^{-2}\cdot(-\sin x)}$

$\sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$

$\dfrac{\pi}{2} = 90^\circ$

$= \lim\limits_{x\to \frac{\pi}{2}} \dfrac{1/\cancel{\cos^2 x}}{\sin x/\cancel{\cos^2 x}} =$

$= \lim\limits_{x\to \frac{\pi}{2}} \dfrac{1}{\sin x} = 1$