MATEMATIKA I

Kalkulatu ondorengo funtzioen debibadak eta laburtu ahalik eta gehien:

1.- $y=\dfrac{3x^4-2x}{5x^5}$ $\qquad y=\left(\dfrac{x^3}{3}\right)^5$ $\qquad y=\dfrac{5\sqrt[5]{x^2}}{x^3}$

$y=\dfrac{3\sqrt[4]{x}-\sqrt[3]{x^2}}{x^4}$ $\qquad y=\dfrac{3x\sqrt{x}+5x^2}{\sqrt[3]{x^2}}$ $\qquad y=\dfrac{\left[(2x)^5\cdot x\right]^3}{\sqrt{x}}$

2.- $y=\dfrac{7x^3+2x}{x^2}$ $\qquad y=\dfrac{7x^3+2x}{1-x^2}$ $\qquad y=\dfrac{x^4-x+1}{e^x+1}$

$y=\dfrac{3\ln x}{2x^3}$ $\qquad y=\dfrac{3senx+x^2}{x^2-2}$ $\qquad y=\dfrac{x^5\cdot 3senx}{x^2-2}$

3.- $y=\left(3x^2-2x\right)^6$ $\qquad y=\sqrt[3]{3x^2-2x}$ $\qquad y=\ln\left(3x^2-2x\right)$

$y=ar\cos\left(3x^2-2x\right)$ $\qquad y=\ln\sqrt[5]{3x^2-2x}$ $\qquad y=e^{3x^2-2x}$

4.- $y=\cos x$ $\qquad y=\cos x^4$ $\qquad y=\cos^4 x^4$

$y=\cos\left(x^4+x^3\right)$ $\qquad y=\ln\left(\cos x^4\right)$ $\qquad y=\sqrt{\cos^4 x}$

$y=\sqrt[3]{\cos x^4}$ $\qquad y=3^{\cos x}$ $\qquad y=3^{\cos x^3}$

5.- $y=\ln x$ $\qquad y=\ln\dfrac{1}{x}$ $\qquad y=\ln 3x^5$

$y=\ln\left(3x^5+5x\right)$ $\qquad y=\ln\left[\left(3x^5+5x\right)\cdot senx\right]$ $\qquad y=\ln\left(3x^5+5x\right)\cdot senx$

$y=\log\dfrac{x^2+3}{tgx}$ $\qquad y=\log 10^{senx^3}$ $\qquad y=\ln\dfrac{x^2+1}{e^x}$

6.- $y=sen^4x\cdot senx^4$ $\qquad y=\dfrac{e^{senx}}{tagx}$ $\qquad y=\dfrac{5^x}{\sqrt{x}}$

7.- $y=arctagx$ $\qquad y=arctag5x^3$ $\qquad y=arctag\left(5x^3+3^x\right)$

1.) $y = \dfrac{3x^4 - 2x}{5x^5} = \dfrac{3}{5}\dfrac{x^4}{x^5} - \dfrac{2x}{5x^5} = \dfrac{3}{5}\dfrac{1}{x} - \dfrac{2}{5}\dfrac{1}{x^4}$

$= \dfrac{3}{5}x^{-1} - \dfrac{2}{5}x^{-4}$

$y' = \dfrac{3}{5}(-1)x^{-2} - \dfrac{2}{5}(-4)x^{-5} = -\dfrac{3}{5x^2} + \dfrac{8}{5x^5} = \boxed{\dfrac{-3x^3+8}{5x^5}}$

$y = (x^3/3)^5 = x^{15}/3^5$

$y' = \dfrac{1}{3^5}15x^{14} = \boxed{\dfrac{15}{243}x^{14}}$

$y = \dfrac{5\sqrt[5]{x^2}}{x^3} = \dfrac{5x^{2/5}}{x^3} = 5x^{2/5-3} = \boxed{5x^{-13/5}}$

$y' = -\dfrac{13}{\cancel{5}}\cancel{5}x^{-\frac{13}{5}-1} = -13x^{-18/5} = \dfrac{-13}{\sqrt[5]{x^{18}}} = \dfrac{-13}{x^3\sqrt[5]{x^3}} = \boxed{\dfrac{-13\sqrt[5]{x^2}}{x^4}}$

$y = \dfrac{3\sqrt[4]{x} - \sqrt[3]{x^2}}{x^4} = \dfrac{3x^{1/4} - x^{2/3}}{x^4} = 3x^{1/4-4} - x^{2/3-4} =$

$y = 3x^{-15/4} - x^{-10/3}$

$y' = 3\cdot\left(-\dfrac{15}{4}\right)\cdot x^{-19/4} - \left(-\dfrac{10}{3}\right)x^{-13/3} = -\dfrac{45}{4\sqrt[4]{x^{19}}} + \dfrac{10}{3\sqrt[3]{x^{13}}}$

$= -\dfrac{45}{4x^4\sqrt[4]{x^3}} + \dfrac{10}{3x^4\sqrt[3]{x}} = \dfrac{-45\sqrt[4]{x}}{4x^4\cdot x} + \dfrac{10\sqrt[3]{x^2}}{3x^4x} = \dfrac{-135\sqrt[4]{x} + 40\sqrt[3]{x^2}}{12x^5}$

$y = \dfrac{\left[(2x)^5\cdot x\right]^3}{\sqrt{x}} = 2^{15}\cdot x^{18-1/2} = 2^{15}\cdot x^{35/2}$

$y' = 2^{15}\cdot\dfrac{35}{2}x^{\frac{35}{2}-1} = 2^{14}\cdot 35\cdot x^{33/2} = 2^{14}35\sqrt{x^{33}} = 2^{14}35x^{16}\sqrt{x}$

$y' = 2^{14}\cdot 35\cdot x^{16}\sqrt{x} = \boxed{573440\, x^{16}\sqrt{x}}$

2] $y = \dfrac{7x^3+2x}{x^2}$

Bi modutara

① $y = 7x + 2/x = 7x + 2x^{-1}$ $\quad y' = 7 + 2\cdot(-1)x^{-2} = 7 - \dfrac{2}{x^2}$

② Zatiketa deribatzen

$$y' = \frac{(21x^2+2)x^2 - (7x^3+2x)2x}{x^4} = \frac{21x^4+2x^2-14x^4-4x^2}{x^4}$$

$$= \frac{7x^4+2x}{x^4} = 7 - \frac{2}{x^2}$$

$$y = \frac{7x^3+2x}{1-x^2} \qquad y' = \frac{(21x^2+2)(1-x^2)-(7x^3+2x)(-2x)}{(1-x^2)^2} =$$

$$= \frac{21x^2-21x^4+2-2x^2+14x^4+4x^2}{(1-x^2)^2} = \frac{-7x^4+23x^2+2}{(1-x^2)^2}$$

$$y = \frac{x^4-x+1}{e^x+1} \qquad y' = \frac{(4x^3-1)(e^x+1)-(x^4-x+1)e^x}{(e^x+1)^2}$$

$$= \frac{4x^3e^x+4x^3-e^x-1-e^xx^4+e^xx-e^x}{(e^x+1)^2} =$$

$$= \frac{e^x(4x^3-2-x^4+x)+4x^3-1}{(e^x+1)^2} = \frac{e^x(-x^4+4x^3+x-2)+4x^3-1}{(e^x+1)^2}$$

$$y = \frac{3\ln x}{2x^3} \qquad y' = \frac{3\frac{1}{x}\cdot 2x^3 - 3\ln x\cdot 6x^2}{(2x^3)^2} = \frac{6x^2-18x^2\ln x}{4x^6}$$

$$= \frac{2x^2(3-9\ln x)}{4x^6} = \frac{3-9\ln x}{2x^4} = \frac{3}{2x^4} - \frac{9}{2x^4}\ln x$$

$$y = \frac{3\sin x + x^2}{x^2-2} \qquad y' = \frac{(3\cos x+2x)(x^2-2)-(3\sin x+x^2)2x}{(x^2-2)^2}$$

$$= \frac{3x^2\cos x - 6\cos x + \cancel{2x^3} - 4x - 6x\sin x - \cancel{2x^3}}{(x^2-2)^2}$$

$$= \frac{\cos x(3x^2-6) - 6x\sin x - 4x}{(x^2-2)^2}$$

$$y = \frac{x^5 \cdot 3\sin x}{x^2-2}$$

$$y' = \frac{(x^5 \cdot 3\sin x)'(x^2-2) - (x^5 \cdot 3\sin x)\cdot(x^2-2)'}{(x^2-2)^2}$$

$$= \frac{(5x^4 \cdot 3\sin x + x^5 \cdot 3\cos x)(x^2-2) - (x^5 \cdot 3\sin x)\cdot 2x}{(x^2-2)^2}$$

$$= \frac{15x^6\sin x - 30x^4\sin x + 3x^7\cos x - 6x^5\cos x - 6x^6\sin x}{(x^2-2)^2}$$

$$= \frac{\sin x(9x^6 - 30x^4) + 3\cos x(x^7-2x^5)}{(x^2-2)^2}$$

$\boxed{3}$ $y = (3x^2-2x)^6$

$$y' = 6(3x^2-2x)^5(6x-2)$$
$$= \underline{(36x-12)(3x^2-2x)^5}$$

$$y = \sqrt[3]{3x^2-2x} = (3x^2-2x)^{1/3}$$

$$y' = \frac{1}{3}(3x^2-2x)^{\frac{1}{3}-1}\cdot(6x-2) =$$

$$= \frac{6x-2}{3}(3x^2-2x)^{-2/3} = \frac{6x-2}{3\sqrt[3]{(3x^2-2x)^2}}$$

$$y = \ln(3x^2-2x)$$

$$y' = \frac{1}{3x^2-2x}(6x-2)$$

$$y = \arccos(3x^2-2x)$$

$$y' = \frac{-1}{\sqrt{1-(3x^2-2x)^2}}(6x-2)$$

$$y = \ln\sqrt[5]{3x^2-2x}$$

$$y' = \frac{1}{\sqrt[5]{3x^2-2x}}\,\frac{1}{5}(3x^2-2x)^{-4/5}(6x-2) = \frac{1}{5}\,\frac{6x-2}{3x^2-2x}$$

$$y = \frac{1}{5}\ln(3x^2-2x)$$

$$\hookrightarrow y' = \frac{1}{5}\,\frac{1}{3x^2-2x}(6x-2)$$

Bi MODUTARA

$y = e^{3x^2-2x}$ $\quad y' = e^{3x^2-2x} \cdot (6x-2)$

$\boxed{4}$

$y = \cos x \quad y' = -\sin x$

$y = \cos x^4 \quad y' = -\sin x^4 \cdot 4x^3 = -4x^3 \cdot \sin x^4$

$y = \cos^4 x^4 = (\cos x^4)^4 \quad y' = 4(\cos x^4)^3 \cdot (-\sin x^4) \cdot 4x^3$
$= -16x^3 \sin x^4 \cdot (\cos x^4)^3$

$y = \cos(x^4+x^3) \longrightarrow y' = -\sin(x^4+x^3) \cdot (4x^3+3x^2)$

$y = \ln(\cos x^4) \to y' = \frac{1}{\cos x^4}(-\sin x^4) \cdot 4x^3 =$
$y' = -tg(x^4) \cdot 4x^3$

$y = \sqrt{\cos^4 x} = (\cos^4 x)^{1/2} = [(\cos x)^4]^{1/2} = (\cos x)^2$
$y' = 2\cos x \cdot (-\sin x) = -\sin(2x)$ $\quad$ $\sin(2x) = 2\sin x \cos x$

$y = \sqrt[3]{\cos(x^4)} = [\cos x^4]^{1/3}$
$y' = \frac{1}{3}[\cos(x^4)]^{1/3-1} \cdot (-\sin(x^4) \cdot 4 \cdot x^3] =$
$y' = \frac{1}{3}[\cos(x^4)]^{-2/3}(-\sin x^4) \cdot 4x^3$
$y' = \frac{-4x^3 \sin x^4}{3\sqrt[3]{(\cos x^4)^2}}$

$y = 3^{\cos x} \quad y' = 3^{\cos x} \ln 3 \, (-\sin x)$
$= -\ln 3 \cdot \sin x \cdot 3^{\cos x}$

$y = 3^{\cos(x^3)} \quad y' = 3^{\cos(x^3)} \ln 3 \cdot (-\sin(x^3)) \cdot 3x^2$
$= -\underbrace{3\ln 3}_{\ln 27} x^2 \sin(x^3) \cdot 3^{\cos(x^3)}$

Bestia $3^{\cos x^3+1}$

5. $y = \ln x \quad y' = 1/x$

$y = \ln 1/x = \cancel{\ln 1}_{0} - \ln x \rightarrow y' = -\frac{1}{x}.$

$y = \ln 3x^5 = \ln 3 + \ln x^5 = \ln 3 + 5\ln x$

$y' = 0 + \frac{5}{x}$

$y = \ln(3x^5 + 5x)$

$y' = \frac{15x^4+5}{3x^5+5x}$

$y = \ln\left[(3x^5+5x)\cdot \sin x\right] = \ln(3x^5+5x) + \ln \sin x$

$y' = \frac{15x^4+5}{3x^5+5x} + \frac{\cos x}{\sin x} = \frac{15x^4+5}{3x^5+5x} + \cot g\, x.$

$y = \underbrace{\ln(3x^5+5x)}_{f} \cdot \underbrace{\sin x}_{g}$

$f' g + f g'$

$y' = \frac{15x^4+5}{3x^5+5x} \cdot \sin x + \ln(3x^5+5x)\cdot \cos x.$

$y = \log \frac{x^2+3}{\operatorname{tg} x} = \log(x^2+3) - \log(\operatorname{tg} x)$

$y' = \frac{2x}{(x^2+3)\ln 10} - \frac{1+\operatorname{tg}^2 x}{\operatorname{tg} x \cdot \ln 10} =$

$y' = \frac{2x\cdot \operatorname{tg} x - (1+\operatorname{tg}^2 x)(x^2+3)}{\ln 10\,(x^2+3)\operatorname{tg} x}$

$y = \log 10^{\sin x^3} = \sin(x^3)\cdot \underbrace{\cancel{\log 10}}_{kte}^{1} = \sin(x^3)$

$y' = \cos(x^3)\cdot 3x^2 = 3x^2\cdot \cos(x^3)$

$$y = \ln \frac{x^2+1}{e^x} = \ln(x^2+1) - \ln e^{x} =$$
$$= \ln(x^2+1) - x \cdot \underbrace{\cancel{\ln e}}_{1} =$$
$$= \ln(x^2+1) - x$$

$$y' = \frac{2x}{x^2+1} - 1 = \frac{2x - x^2 - 1}{x^2+1} = \boxed{-\frac{(x-1)^2}{x^2+1}}$$

6/

$$y = \underbrace{\sin^4 x}_{f} \cdot \underbrace{\sin(x^4)}_{g} = (\sin x)^4 \cdot \sin(x^4)$$

$$y' = \underbrace{4(\sin x)^3 \cdot \cos x}_{f'} \cdot \underbrace{\sin(x^4)}_{g} + \underbrace{(\sin x)^4}_{f} \cdot \underbrace{\cos(x^4) \cdot 4x^3}_{g'}$$

$$y' = 4(\sin x)^3 \left[\cos x \sin(x^4) + x^3 \cdot \sin x \cdot \cos(x^4)\right]$$

$$y = \frac{e^{\sin x}}{\operatorname{tg} x}$$

$$y' = \frac{\underbrace{e^{\sin x} \cdot \cos x}_{f'} \cdot \underbrace{\operatorname{tg} x}_{g = \frac{\sin}{\cos}} - \underbrace{e^{\sin x}}_{f} \cdot \underbrace{\frac{1}{\cos^2 x}}_{g'}}{\underbrace{\left(\frac{1}{\cos^2 x}\right)^2}_{g^2}} =$$

$$= \frac{e^{\sin x} \cdot \sin x - e^{\sin x} \frac{1}{\cos^2 x}}{\left(1/\cos^2 x\right)^2}$$

$$= e^{\sin x} \frac{\frac{\sin x \cdot \cos^2 x - 1}{\cancel{\cos^2 x}}}{1/\cancel{\cos^4 x}_{\cos^2 x}}$$

$$= e^{\sin x} (\sin x \cdot \cos^2 x - 1) \cdot \cos^2 x$$

$$y = \frac{5^x}{\sqrt{x}} \qquad y' = \frac{5^x \ln 5 \cdot \sqrt{x} - 5^x \frac{1}{2\sqrt{x}}}{x} =$$

$$= \frac{\dfrac{5^x \ln 5\, 2x\sqrt{x} - 5^x\sqrt{x}}{2x.}}{x} = \frac{5^x(\ln 5\, 2x\sqrt{x} - \sqrt{x})}{2x^2}$$

$$= \frac{5^x\sqrt{x}\,(2\ln 5\, x - 1)}{2x^2}$$

$$y = \operatorname{artg} x \qquad y' = \frac{1}{1+x^2}$$

$$y = \operatorname{artg}(5x^3) \qquad y' = \frac{1}{1+(5x^3)^2} \cdot 15x^2 = \frac{15x^2}{1+25x^6}.$$

$$y = \operatorname{artg}(5x^3 + 3^x)$$

$$y' = \frac{1}{1+(5x^3+3^x)^2} \cdot (15x^2 + 3^x \ln 3).$$