350) **13** a) $\int \underset{P}{x^2} \cdot \underset{S}{\sin x}\, dx$

$$\begin{cases} u = x^2 \longrightarrow du = 2x \cdot dx \\ dv = \sin x \cdot dx \longrightarrow v = -\cos x \end{cases}$$

$I = x^2 \cdot (-\cos x) - \int (-\cos x) \cdot 2x\, dx =$

$I = -x^2 \cos x + 2 \underbrace{\int \cos x \cdot x\, dx}_{I_1}$ $\qquad I_1 \begin{cases} u = x \to du = dx \\ dv = \cos x \to v = \sin x \end{cases}$

$I_1 = x \cdot \sin x - \int \sin x \cdot dx = x \cdot \sin x - (-\cos x)$

$$\boxed{I = -x^2 \cos x + 2 \cdot (x \cdot \sin x + \cos x) + K}$$

b) $\int \underset{P}{x^2} \underset{E}{e^{2x}} dx$ $\qquad \begin{cases} u = x^2 \longrightarrow du = 2x\, dx \\ dv = e^{2x} dx \longrightarrow v = \frac{1}{2}\int 2e^{2x} dx = \frac{1}{2} e^{2x} \end{cases}$

$I = x^2 \cdot \frac{1}{2} e^{2x} - \int \frac{1}{\cancel{2}} e^{2x} \cdot \cancel{2}x \cdot dx$

$I = \frac{x^2 \cdot e^{2x}}{2} - \underbrace{\int x \cdot e^{2x} dx}_{I_1}$ $\qquad I_1 \begin{cases} u = x \to du = dx \\ dv = e^{2x} dx \to v = \frac{1}{2} e^{2x} \end{cases}$

$I_1 = x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} \cdot dx = \frac{x \cdot e^{2x}}{2} - \frac{1}{2} \frac{1}{2} e^{2x}$

$I = \frac{x^2 \cdot e^{2x}}{2} - I_1$

$I = \frac{x^2 \cdot e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{1}{4} e^{2x} = \frac{e^{2x}}{2} (x^2 - x + \frac{1}{2}) + K$

c) $\int \underset{E}{e^x} \underset{S}{\cos x}\, dx \quad \left| \begin{array}{l} u = e^x \longrightarrow du = e^x dx \\ dv = \cos x\, dx \rightarrow v = \sin x \end{array} \right.$

$$I = e^x \cdot \sin x - \underbrace{\int \underset{S}{\sin x} \cdot \underset{E}{e^x}\, dx}_{I_1} \quad \left| \begin{array}{l} u = e^x \rightarrow du = e^x dx \\ dv = \sin x\, dx \rightarrow v = -\cos x \end{array} \right.$$

$$I_1 = e^x \cdot (-\cos x) - \int -\cos x \cdot e^x\, dx$$

$$= -e^x \cos x + \underbrace{\int \cos x\, e^x\, dx}_{I}$$

$$\int e^x \cos x\, dx = e^x \cdot \sin x - \left(-e^x \cos x + \int \cos x \cdot e^x\, dx\right)$$

$$\underbrace{\int e^x \cos x\, dx}_{} = e^x \sin x + e^x \cos x - \underbrace{\int \cos x \cdot e^x\, dx}_{}$$

$$I = e^x \sin x + e^x \cos x - I$$

$$2I = e^x \sin x + e^x \cos x$$

$$\boxed{I = \frac{e^x (\sin x + \cos x)}{2}}$$