MATEMATIKA II	2.EBALUAKETA – PARTZIALA	2022/12/20	
IZEN-ABIZENAK:______________________________		TALDEA: _____	
Planteamenduak egokiak izan behar dira, kontzeptuak, hizkuntza eta notazio zientifikoa zuzenak izan behar dira eta egindako urratsen azalpen garbia eta aurkezpena txukuna izan beharko dira.			

1. Kalkulatu hurrengo berehalako integral mugagabeak: (2,5p)

a) $\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{9(1-\frac{x^2}{9})}} = \int \frac{dx}{3\sqrt{1-(\frac{x}{3})^2}} = \frac{1}{3} \frac{1}{1/3} \int \frac{1/3\, dx}{\sqrt{1-(x/3)^2}} =$

$= \boxed{\arcsin\left(\frac{x}{3}\right) + K}$

b) $\int (2x-4)\sqrt[3]{x^2-4x}\, dx = \int (2x-4)\cdot(x^2-4x)^{1/3} dx = \frac{(x^2-4x)^{1/3+1}}{1/3+1} + K =$

$= \frac{(x^2-4x)^{4/3}}{4/3} + K = \frac{3}{4} \cdot \sqrt[3]{(x^2-4x)^4} + K = \boxed{\frac{3}{4}(x^2-4x)\sqrt[3]{x^2-4x} + K}$

c) $\int sin(3x)cos(3x)\, dx = \int u \cdot \frac{du}{3} = \frac{1}{3}\int u\, du = \frac{1}{3}\frac{u^2}{2} + K =$

$u = \sin(3x)$

$du = 3\cdot\cos(3x)dx$

$\frac{du}{3} = \cos(3x)dx$

$= \boxed{\frac{[\sin(3x)]^2}{6} + K}$

EDO BEREHALAKOA:

$I = \frac{1}{2}\cdot\frac{1}{3}\int 2\cdot 3 \sin(3x)\cdot\cos(3x)\cdot dx = \boxed{\frac{1}{6}\sin^2(3x) + K}$

d) $\int \frac{x}{8-4x^2} dx =$

$= -\frac{1}{8}\int \frac{-8x}{8-4x^2} dx = \boxed{-\frac{1}{8} Ln|8-4x^2| + K_1}$

K desberdinak

EDO: $I = \int \frac{x}{4(2-x^2)} dx = \frac{1}{4}\left(-\frac{1}{2}\right)\int \frac{-2x}{2-x^2} dx = \boxed{-\frac{1}{8} Ln|2-x^2| + K_2}$

e) $\int \frac{\sqrt{7}dx}{5+35x^2} = \int \frac{\sqrt{7}\, dx}{5(1+7x^2)} = \frac{1}{5}\int \frac{\sqrt{7}\, dx}{1+(\sqrt{7}x)^2} =$

$= \boxed{\frac{1}{5} arctg(\sqrt{7}x) + K}$

2. Kalkulatu ondorengo integral mugagabea <u>ordezkapen-metodoa</u> erabiliz: **1p**

$$\int \frac{2e^x}{\sqrt{e^x+1}}dx$$

ORDEZKAPEN METODOA

$e^x+1 = t^2 \longrightarrow t = \sqrt{e^x+1}$

$e^x dx = 2t\,dt$

$$\int \frac{2e^x dx}{\sqrt{e^x+1}} = \int \frac{2 \cdot 2t\,dt}{\sqrt{t^2}} = \int \frac{4\cancel{t}\,dt}{\cancel{t}} =$$

$$= 4\int dt = 4t + K = \boxed{4\sqrt{e^x+1} + K}$$

3. Kalkulatu ondorengo integral mugagabeak: **(1,25+1,75)** **3p**

$$\int \frac{x^4-9x^2+2}{x-3}dx$$

P(x)-ren maila > Q(x) maila, beraz
ZATIKETA egiten da:

$$\begin{array}{rrrr|l} x^4 & & -9x^2 & +2 & x-3 \\ -x^4 & +3x^3 & & & x^3+3x^2 \\ \hline / & 3x^3 & -9x^2 & +2 & \\ & -3x^3 & +9x^2 & & \\ \hline & / & / & 2 & \end{array}$$

$$I = \int \left(x^3 + 3x^2 + \frac{2}{x-3} \right) dx = \boxed{\frac{x^4}{4} + \frac{3x^3}{3} + 2\ln|x-3| + K}$$

$$\int \frac{x^2+7}{x^3-2x^2+x}\,dx = \int \frac{x^2+7}{x(x-1)^2}\,dx$$

Erro ~~sinpleak~~

$x=0$

$x=1$ BIKOITZA.

$$\frac{x^2+7}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x-1)\cdot x + C\cdot x}{x(x-1)^2}$$

$$x=0 \longrightarrow 7 = A + \cancel{(-B)\cdot 0} + \cancel{C\cdot 0} \longrightarrow \boxed{A=7}$$

$$x=1 \longrightarrow 1+7 = \cancel{A\cdot 0} + \cancel{B\cdot 0} + C \longrightarrow \boxed{C=8}$$

$$x=-1 \longrightarrow 1+7 = 4A + 2B - C$$

$$8 = 4\cdot 7 + 2\cdot B - 8 \longrightarrow \boxed{B=-6}$$

$$I = \int\left(\frac{7}{x} + \frac{-6}{x-1} + \frac{8}{(x-1)^2}\right)dx =$$

$$= 7\ln|x| - 6\ln|x-1| + 8\cdot\frac{(x-1)^{-2+1}}{-2+1} + K$$

$$\boxed{I = 7\ln|x| - 6\ln|x-1| - \frac{8}{x-1} + K}$$

4. Kalkulatu integral mugagabe hauek: ZATIKAKO METODOA **(1+2) 3p**

$$\int x\ln(4x)\,dx =$$

$$\begin{cases} u = \ln(4x) \longrightarrow du = \frac{\cancel{4}}{\cancel{4}x}\,dx = \frac{1}{x}\,dx \\ dv = x\cdot dx \longrightarrow v = \int x\,dx = \frac{x^2}{2} \end{cases}$$

$$I = u\cdot v - \int v\cdot du$$

$$I = \ln(4x)\cdot\frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\cdot dx =$$

$$= \ln(4x)\cdot\frac{x^2}{2} - \int\frac{x}{2}\cdot dx = \ln(4x)\cdot\frac{x^2}{2} - \frac{1}{2}\int x\,dx =$$

$$= \frac{x^2}{2}\cdot\ln(4x) - \frac{1}{2}\cdot\frac{x^2}{2} + K = \boxed{\frac{x^2}{2}\cdot\left[\ln(4x) - \frac{1}{2}\right] + K}$$

ZATIKAKO METODOA

$I = \int \underset{P}{x^2} \underset{E}{e^{5x}}\, dx =$

$$I = \begin{cases} \boxed{u = x^2} \longrightarrow du = 2x\,dx \\ dv = e^{5x}dx \longrightarrow v = \int e^{5x}dx = \frac{1}{5}\int 5e^{5x}dx = \boxed{\frac{1}{5}e^{5x}} \end{cases}$$

$$I = u \cdot v - \int v \cdot du = x^2 \cdot \frac{1}{5} \cdot e^{5x} - \int \frac{1}{5} e^{5x} \cdot 2x\,dx =$$

$$= \frac{1}{5}x^2 \cdot e^{5x} - \frac{2}{5}\underbrace{\int e^{5x} \cdot x\,dx}_{I_1}$$

$$I_1 \begin{cases} u = x \longrightarrow du = dx \\ dv = e^{5x}dx \longrightarrow v = \frac{1}{5}e^{5x} \end{cases}$$

$$I_1 = x \cdot \frac{1}{5}e^{5x} - \int \frac{1}{5}e^{5x} \cdot dx = \frac{x}{5}e^{5x} - \frac{1}{5}\frac{1}{5}\int 5e^{5x}dx$$

$$I_1 = \frac{x}{5}e^{5x} - \frac{1}{25}e^{5x}$$

$$I = \frac{1}{5}x^2 \cdot e^{5x} - \frac{2}{5}I_1 = \frac{1}{5}x^2 \cdot e^{5x} - \frac{2}{5}\left[\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x}\right] + K$$

$$\boxed{I = \frac{1}{5}e^{5x}\left[x^2 - \frac{2}{5}x + \frac{2}{25}\right) + K.}$$

5. Aukeratu BIETARIKO bat 0,5p

1) $\int \frac{\sqrt{x}-1}{\sqrt[3]{x}}\,dx =$

2) $\int \frac{dx}{x^2 - 6x + 10} =$

1) $$\int \frac{\sqrt{x}-1}{\sqrt[3]{x}}\,dx = \int \frac{\sqrt{x}}{\sqrt[3]{x}} - \frac{1}{\sqrt[3]{x}}\,dx = \int (x^{\frac{1}{2}-\frac{1}{3}} - x^{-1/3})\,dx =$$

$$= \int x^{\frac{3-2}{6}} - x^{-1/3}\,dx = \int (x^{1/6} - x^{-1/3})\,dx = \frac{x^{1/6+1}}{1/6+1} - \frac{x^{-1/3+1}}{-1/3+1} + K =$$

$$= \frac{x^{7/6}}{7/6} - \frac{x^{2/3}}{2/3} + K = \frac{6}{7}\sqrt[6]{x^7} - \frac{3}{2}\sqrt[3]{x^2} + K =$$

$$= \boxed{\frac{6}{7}x\sqrt[6]{x} - \frac{3}{2}\sqrt[3]{x^2} + K}$$

① Besto modu bat.

ORDEZKAPEN METODOA

$x = t^6 \longrightarrow t = \sqrt[6]{x}$

$dx = 6t^5 dt$

B

$$\int \frac{\sqrt{x}-1}{\sqrt[3]{x}}\,dx =$$

$$= \int \frac{\sqrt{t^6}-1}{\sqrt[3]{t^6}}\,6t^5 dt = \int \frac{t^3-1}{\cancel{t^2}}\cdot 6t^{\cancel{5}\,3} dt =$$

$$= 6\int (t^3-1)\,t^3\,dt = 6\int (t^6-t^3)\,dt =$$

$$= 6\left(\frac{t^7}{7}-\frac{t^4}{4}\right)+K = 6\left(\frac{(\sqrt[6]{x})^7}{7}-\frac{(\sqrt[6]{x})^4}{4}\right)+K$$

$$= 6\left(\frac{\sqrt[6]{x^7}}{7}-\frac{\sqrt[6]{x^4}}{4}\right)+K = \boxed{6\left(\frac{x\sqrt[6]{x}}{7}-\frac{\sqrt[3]{x^2}}{4}\right)+K.}$$

② $$\int \frac{dx}{x^2-6x+10} =$$

$$\underbrace{x^2-6x+9}_{(x-3)^2}\underbrace{-9+10}_{+1}$$

$$= \int \frac{dx}{x^2-6x+9-9+1} = \int \frac{dx}{(x-3)^2+1} = \boxed{\operatorname{arctg}(x-3)+K}$$