2024 EL OHIKOA

A4 $\int x \cdot \ln^2 x \, dx$

$P \quad L$

$\begin{cases} u = \ln^2 x \rightarrow du = \frac{2\ln x}{x} dx \\ dv = x\,dx \rightarrow v = \int x\,dx = \frac{x^2}{2} \end{cases}$

$uv - \int v\,du$

$I = \ln^2 x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{2\ln x}{x} dx =$

$= \frac{x^2}{2} \cdot \ln^2 x - \underbrace{\int x \ln x \, dx}_{I_1}$ $\quad P \quad L$

$I_1 = \begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = x\,dx \rightarrow v = \frac{x^2}{2} \end{cases}$

$I_1 = \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \cdot \ln x - \frac{1}{2} \frac{x^2}{2}$

$I = \frac{x^2}{2} \cdot \ln^2 x - \left[\ln x \cdot \frac{x^2}{2} - \frac{1}{2} \frac{x^2}{2} \right] + k$

$I = \frac{1}{2} \cdot x^2 \ln^2 x - \frac{x^2}{2} \cdot \ln x + \frac{1}{4} x^2 + k$

$$\boxed{I = \frac{x^2}{2} \left(\ln^2 x - \ln x + \frac{1}{2} \right) + k}$$