AZALERA

$f(x) = x^3 - 6x^2 + 9x$

↳ Kurbak eta OX ardatzak mugatutako eskualdearen azalera.

① Adierazi kurba grafikoki

• $\text{Dom} f = \mathbb{R}$

• $x = 0 \Rightarrow P(0,0)$

$y = 0 \Rightarrow x^3 - 6x^2 + 9x = 0$

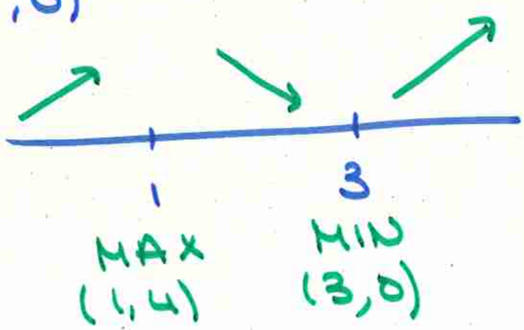
$x(x^2 - 6x + 9) = 0 \quad P(0,0)$

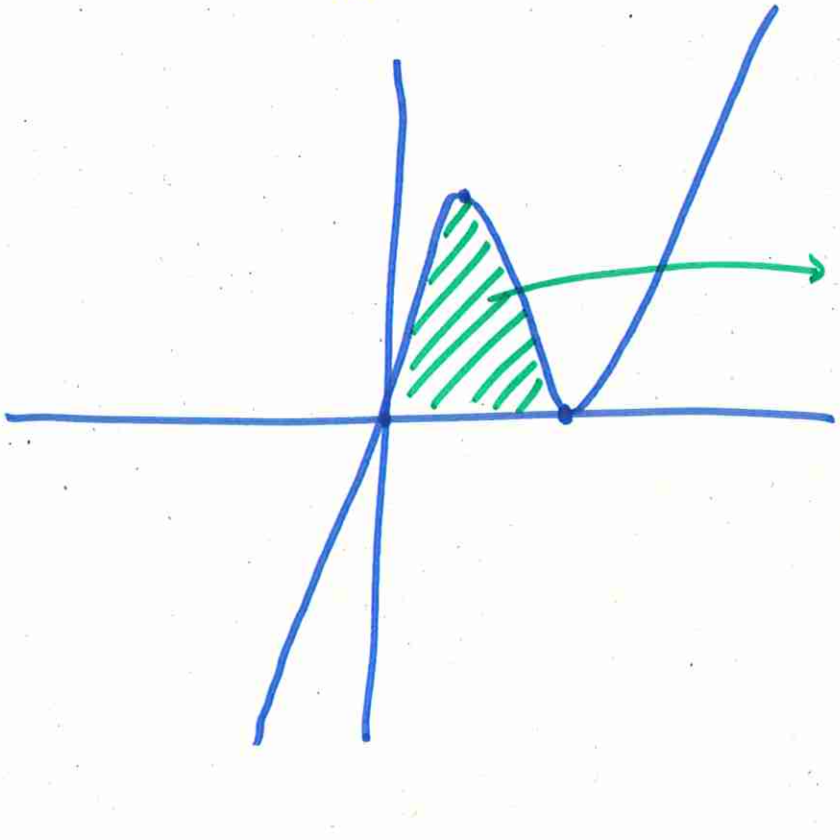
$x(x-3)^2 = 0 \Rightarrow P(3,0)$

• $f'(x) = 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0 \left\langle \begin{array}{l} x = 1 \\ x = 3 \end{array} \right.$

$(x-1)(x-3) = 0$





② Eskualdea adierazi

③ Ebaki puntuak lortu:

$$\left.\begin{array}{l} f(x) = x^3 - 6x^2 + 9x \\ OX \text{ ardatza} \end{array}\right\} \longrightarrow \begin{array}{c} x^3 - 6x^2 + 9x = 0 \\ x = 0 \ , \ x = 3 \end{array}$$

④ Azalera integral mugatuaren bidez:

$$A = \int_0^3 f(x)\,dx = \int_0^3 (x^3 - 6x^2 + 9x)\,dx$$

4.1 → Jatorrizkoa lortu:

$$\int (x^3 - 6x^2 + 9x)\,dx = \int x^3 dx + \int -6x^2 dx + \int 9x\,dx =$$

$$= \frac{x^4}{4} - 6 \cdot \frac{x^3}{3} + 9\frac{x^2}{2} = \frac{x^4}{4} - 2x^3 + \frac{9x^2}{2} + K$$

4.2 → Mugak ordezkatu:

$$A = \left[\frac{x^4}{4} - 2x^3 + \frac{9x^2}{2}\right]_0^3 =$$

¡Azaleran "K" ezeztatzen da.!

$$= \left(\frac{3^4}{4} - 2 \cdot 3^3 + \frac{9 \cdot 3^2}{2}\right) - \left(\frac{0^4}{4} - 2 \cdot 0^3 + \frac{9 \cdot 0^2}{2}\right) =$$

$$= \frac{27}{4} = \boxed{6'75\,u^2}$$

Azaleraren unitatea

AZALERA

$f(x) = -x(x-4) = -x^2 + 4x$

↳ f funtzioak eta OX ardatzak mugaturiko azalera kalkulatu.

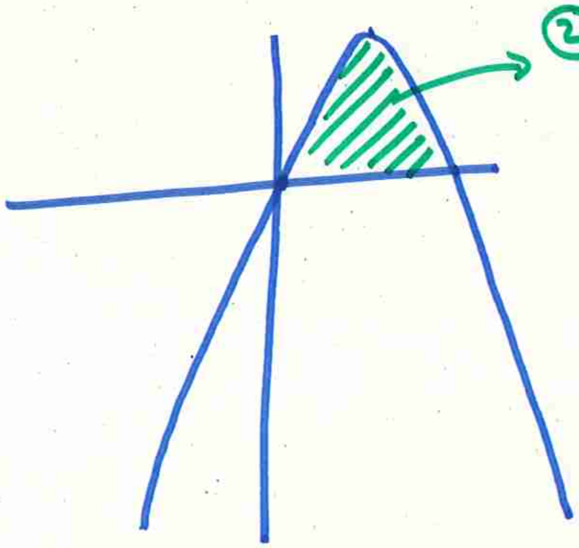
① Adierazi kurba grafikoki

↳ Parabola da, erpina eta balio taula.

- $\text{Dom } f = \mathbb{R}$
- $x = 0 \Rightarrow P(0,0)$

  $y = 0 \Rightarrow P(0,0)$; $P(4,0)$
- $\cancel{f'(x)}$. Erpina: $E_x = \dfrac{-4}{-2} = 2 \Rightarrow E(2, \ )$

x	y
-2	-12
0	0
2	4
4	0
6	-12



② Eskualdea adierazi

③ Ebaki-puntuak:

$$\left.\begin{array}{l} f(x) = -x(x-4) \\ \text{OX ardatza} \end{array}\right\} \rightarrow \begin{array}{c} -x(x-4) = 0 \\ x = 0 \ , \ x = 4 \end{array}$$

④ Azalera integral mugatuaren bidez:

$$A = \int_0^4 f(x)\,dx = \int_0^4 -x(x-4)\,dx$$

4.1 → Jatorrizkoa kalkulatu:

$$\int -x(x-4)\,dx = \int (-x^2+4x)\,dx = -\int x^2\,dx + \int 4x\,dx =$$

$$= -\frac{x^3}{3} + 4\cdot\frac{x^2}{2} = -\frac{x^3}{3} + 2x^2 + K$$

4.2 → Mugak ordezkatu:

$$A = -\frac{x^3}{3} + 2x^2 \Big]_0^4 = -\frac{4^3}{3} + 2\cdot 4^2 = \boxed{\frac{32}{3}\,u^2}$$

AZALERA

$f(x) = x^3 - 4x^2 + 3x$

↳ Funtzioak OX ardatzarekiko sortzen duen eskualdearen azalera

① Adierazi kurba grafikoki

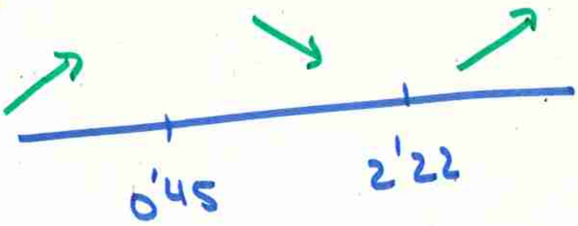
- $\text{Dom}\, f = \mathbb{R}$
- $x = 0 \Rightarrow P(0,0)$

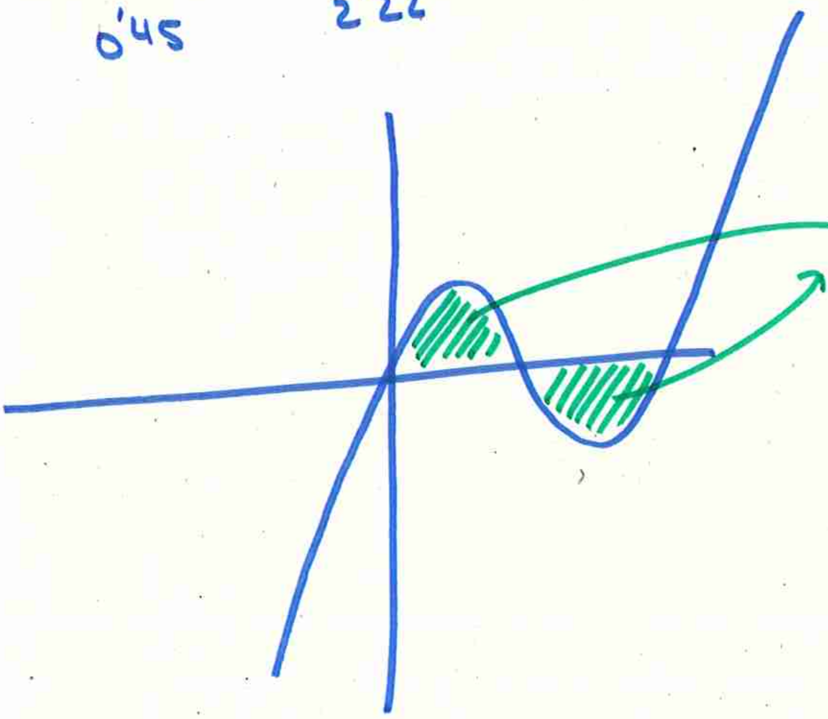
  $y = 0 \Rightarrow x^3 - 4x^2 + 3x = 0$

  $x(x^2 - 4x + 3) = 0$ → $P(0,0)$, $P(1,0)$, $P(3,0)$

- $f'(x) = 3x^2 - 8x + 3 = 0$ → $x \approx 0'45$, $x \approx 2'22$

→ Hau liburutik hartu dut. Selektibitateko emaitzak "politak" dira.





② Eskualdea adierazi

③ Ebaki-puntuak lortu:

$$\left.\begin{array}{l} f(x)=x^3-4x^2+3x \\ OX \text{ ardatza} \end{array}\right\} \begin{array}{l} x^3-4x^2+3x=0 \\ x=0,\ x=1,\ x=3 \end{array}$$

! Bi eremu desberdinduko ditugu:

$\boxed{I}$ [0, 1] tartea

$\boxed{II}$ [1, 3] tartea

④ Azalera integral mugatuaren bidez:

$$A = A_1 + A_2 \begin{cases} A_1 = \int_0^1 f(x)\,dx \\ A_2 = \int_1^3 - f(x)\,dx \end{cases}$$

OX ardatzaren azpian dagoelako.

! $A \neq \int_0^4 f(x)\,dx$

4.1 → Jatorrizkoa lortu:

$$\int (x^3 - 4x^2 + 3x)\,dx = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2} + K$$

$$\int (-x^3 + 4x^2 - 3x)\,dx = -\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} + K$$

4.2 → Mugak ordezkatu:

$$A_1 = \int_0^1 (x^3 - 4x^2 + 3x)\,dx = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\Big]_0^1 =$$

$$= \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right) - \left(\frac{0}{4} - \frac{4\cdot 0}{3} + \frac{3\cdot 0}{2}\right) =$$

$$= \frac{1}{4} - \frac{4}{3} + \frac{3}{2} = \frac{5}{12}\,u^2$$

$$A_2 = \int_1^3 (-x^3 + 4x^2 - 3x)\,dx = -\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2}\Big]_1^3 =$$

$$= \left(-\frac{3^4}{4} + \frac{4\cdot 3^3}{3} - \frac{3\cdot 3^2}{2}\right) - \left(-\frac{1}{4} + \frac{4}{3} - \frac{3}{2}\right) =$$

$$= \frac{9}{4} - \left(-\frac{5}{12}\right) = \frac{8}{3}\,u^2$$

$$A = A_1 + A_2 = \boxed{\frac{37}{12}\,u^2}$$